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A METHOD FOR COMPUTING THE RESPONSE OF CRACKED GRAVITY DAMS TO EARTHQUAKE FORCES

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INTRODUCTION

The Koyna Earthquake of December 11, 1967, caused the Koyna Dam to develop horizontal cracks both on the upstream and downstream faces in a number of monoliths[1, 10]. The cracks were especially severe at elevations where abrupt change occurred in the slope of the downstream face.

Cracks in dams create geometric nonlinearities, even though material properties are assumed to remain linear. Thus, the response problem becomes nonlinear. The normal mode super-position approach becomes difficult, and a direct numerical integration of the equations of motion provides a more attractive approach. For this study the numerical integration was performed using a fourth order Runge-Kutta method.

In this paper a method is developed for formulating an approximate mathematical model of a cracked dam, and the behavior of Koyna Dam under earthquake forces is investigated.

The dam is assumed to have a single horizontal crack extending from the upstream face to the midpoint of the cross-section of the monolith at the elevation of the change in slope of the downstream face, as shown in Fig. 1.

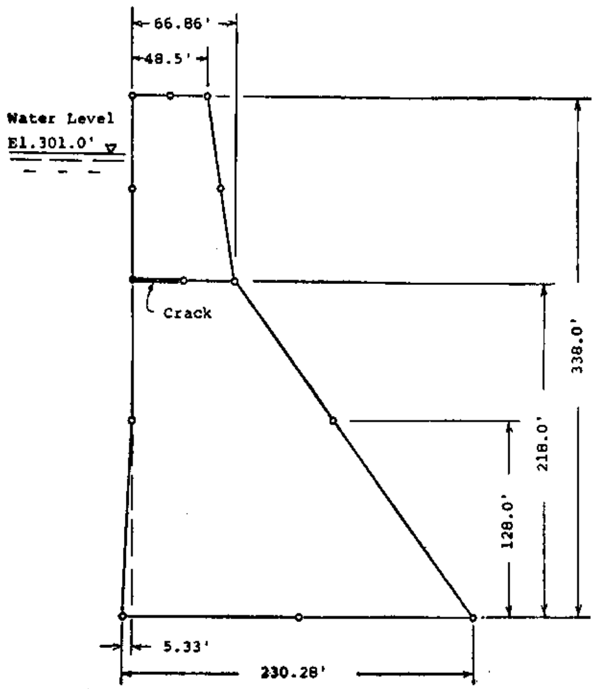

Fig. 1. Finite Element Model of Dam and Crack Location.

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The dimensions shown in Fig. 1 are those of the tallest monolith of Koyna Dam, and the location and extent of the crack are approximately the same as the worst crack caused in that monolith by the earthquake. The behavior of an uncracked section of the same dimension was evaluated, using various values of damping, with and without the virtual mass of the reservoir water. The models were subjected to the El Centro and Koyna ground motion, and the results are compared.

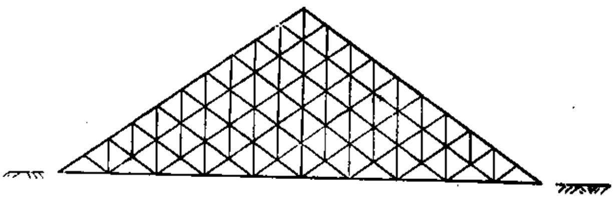
The numerical work utilizes the finite element technique and employs isoparametric elements[2, 3] to obtain the stiffness matrix for the structure. The crack may be represented by linkage elements[4, 5] to accommodate independently the horizontal and vertical stiffnesses across the cracked section. In this particular case it was simpler to compute complete stiffness matrices for the cracked and uncracked sections, and call for whichever one is appropriate at the particular instant.

MATHEMATICAL MODEL OF DAM

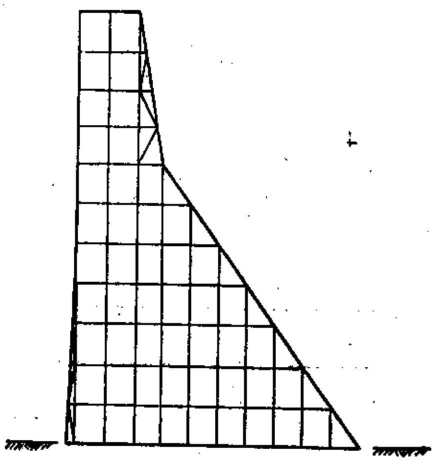
The following assumptions are made in the analysis :

1. The dam is considered to behave in plane strain.
2. The dam is made of homogeneous and isotropic material and is linearly elastic in material behavior.
3. The friction force in the crack is neglected, whether the crack is in the open or closed position.

A constant–stress finite element model for a dam requires subdividing the dam cross-section into many elements and nodal points. Chopra and Clough[6] used 100 elements and 66 nodal points as shown in Fig. 2 (a) in an earthdam study. The resulting number



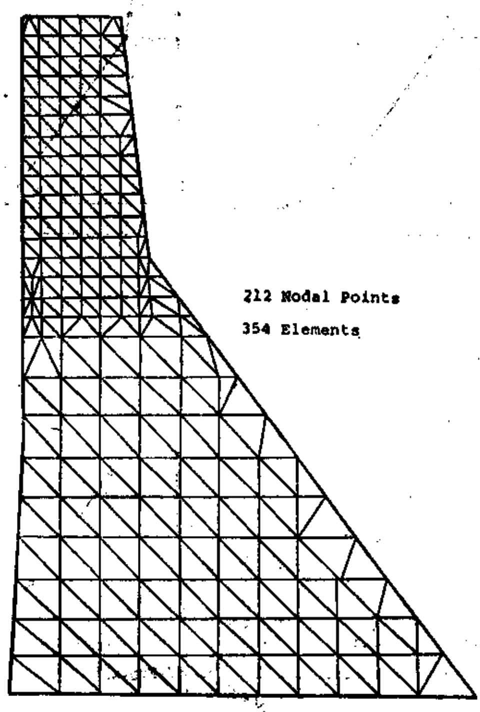
(a) Idealization of Earth Dam - Chopra and Clough[6]



(b) Koyna Dam Idealization - Saini[7]

Fig. 2. Finite Element Idealization of Dams by Various Authors.

of degrees of freedom for the dam was 110. Saini[7] analyzed the Koyna Dam using 70 elements and 79 nodal points as shown in Fig. 2 (b). His model had 134 degrees of



(c) Koyna Dam Idealization for Constant Stress

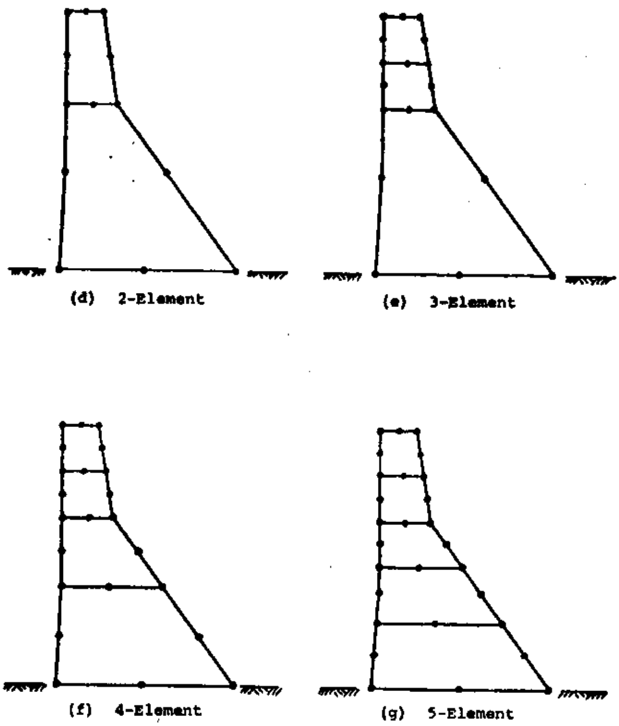
Fig. 2. Finite Element Idealization of Dams by Various Authors.

freedom. This type of subdivision is indeed well suited when normal mode super-position approach is used to solve the dam structure under dynamic loading. One can find the frequencies of the structure, and using some modal superposition method obtain the response of the structure from the first few modes. This was the procedure followed by Chopra and Clough and Saini.

If one were to solve 110 or 134 equations of motions simultaneously using a numerical integration method, the computer time would become excessive. This hurdle was overcome by the use of two-dimensional isoparametric finite elements of 8 nodal points per element[3]. Two elements of this type, arranged as shown in Fig. 1, were sufficient to yield acceptable accuracy.

One indication of the adequacy of this model is given in Tables I and II, which show the computed static load deflections of the uncracked Koyna monolith using 354 constant-stress triangular elements (as used by Clough and Chopra) and 5, 4, 3, and 2 isoparametric elements. The subdivision of the section into elements is shown in Fig. 2 (c to g). The variation in the displacements between 354 constant-stress-element model and 2-isoparametric-element model was less than 4 percent.

The 2-isoparametric-element model is chosen as the mathematical model for this study. It has 14 nodal points and 22 degrees of freedom. It is also well suited to accommodate the horizontal crack in the dam, see Fig. 1.



Isoparametric Idealization of Koyna Dam

Fig. 2. Finite Element Idealization of Dams by Various Authors.

The maximum difference in the extreme vertical stresses across El. 218 ft. (Fig. 1) between the 354 constant-stress-element and 2-isoparametric-element models was less than 12.3 percent. Table II gives further stress comparisons. The stress calculations may be refined if desired. At a given instant in the dynamic problem, for example, the displacement pattern obtained from the analysis can be imposed statically on a model of the dam with many elements, i.e. using a refined mesh, and the stresses may be calculated. This method has been found to be successful.

NUMERICAL WORK

The location and size of the horizontal crack in this model is such that only two states of global stiffness are possible. In the cracked state, vertical and horizontal separation may exist in the crack. In the uncracked state horizontal separation may exist, since it is assumed that there is no friction between the cracked surfaces, but vertical separation may not exist. Thus whether the dam is in the uncracked or cracked state depends entirely on the vertical displacements of nodal points 6 and 7 (see Fig. 1).

Because the global stiffness matrix is not large (22×22), it was simpler to compute and store the complete stiffness matrices for the cracked and uncracked states than to revise the stiffness matrix each time a change of state occurred. During the calculation the appropriate stiffness was called upon during each incremental time step.

Two criteria were applied to determine which stiffness state to use. For the cracked state the vertical displacement of nodal point 6 (upper point) had to be greater than that

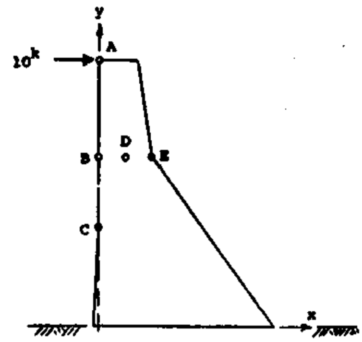


Table I Comparison of Displacements for Various Subdivision and Type of Element Under Static Loading.

Point	Displ–X	Displ–Y	No. of Elements	Element Type	
A	.3362"	.1002"	354	Constant Stress	Triangular
	.3431	.1030	5	Isoparametric	8 Nodal Points
	.3394	.1021	4	”	”
	.3218	.1011	3	”	”
	.3237	.0984	2	”	”
B	.0843	.0605	354	Constant Stress	Triangular
	.0875	.0623	5	Isoparametric	8 Nodal Point
	.0862	.0615	4	”	”
	.0820	.0617	3	”	”
	.0809	.0602	2	”	”
C	.0214	.0265	354	Constant Stress	Triangular
	.0219	.0270	5	Isoparametric	8 Nodal Points
	.0216	.0279	4	”	”
	.0215	.0251	3	”	”
	.0213	.0245	2	”	”

(b)

Table II Comparison of Vertical Stresses for Various Subdivision and Type of Element Under Static Loading. (See Table I for location of points).

Vertical Stress at				
Point B	Point D (mid point)	Point E	Element Type	No. of Elements
117.94	– 4.00	–136.96	Cons. Stress	354
126.96	– 1.76	–137.90	Iso–8	5
124.65	– 8.50	–134.88	Iso–8	4
128.94	– 9.12	–122.71	Iso–8	3
131.06	–10.84	–120.11	Iso–8	2

(Stresses are in psi)

of nodal point 7 (lower point). Otherwise the uncracked state took over. On the other hand if the vertical stress at nodal points 6 and 7 changed from compression to tension, the stiffness was changed to the cracked state.

MASS DISTRIBUTION

Unlike the constant-stress triangular element (where the mass is divided equally among the three nodal points), the 8-nodal-point isoparametric element has an uneven distribution of its mass. If the element is approximately square in shape, the mass is distributed 1/36 to each corner node and 8/36 to each midpoint node. This distribution rule was reached indirectly, and somewhat intuitively. When a uniformly distributed normal load is applied to one edge of a rectangular isoparametric element, the equivalent nodal point forces are 1/6 of the load at the corners and 4/6 at the midpoint. This distribution of load to the nodal points results in uniform stress distribution in the element. A similar reasoning led to the above mass distribution.

VIRTUAL MASS OF RESERVOIR WATER

Following Westergaard's analysis[8], the virtual mass of the water has been represented as an equivalent width of concrete

$$b' = \frac{7}{8} \frac{W_w}{W_c} \sqrt{hy} \qquad \ldots(1)$$

that contributes to the horizontal inertia of the dam but not to its vertical inertia or its stiffness.

In Eq. (1), W_w=density of the water, and

W_c=density of concrete (rubble concrete).

For this study we took h to be 301 ft., which was the height of reservoir water above the base at time of the earthquake, W_w to be 62·4 lbs/cu. ft., and W_c to be 165 lbs/cu. ft.

COEFFICIENT OF DAMPING

A diagonal damping matrix was computed that would give approximately 5 percent of critical damping in each mode for the uncracked state.

The equations of motion for free vibration are

$$M\ddot{x} + C\dot{x} + Kx = 0 \qquad \ldots(2)$$

where M= inertia matrix

C=damping matrix

K=stiffness matrix

x=displacement vector.

The damping matrix may be assumed as

$$C = 2M\phi \begin{bmatrix} \diagdown & & \\ & \beta & \\ & & \diagdown \end{bmatrix} \begin{bmatrix} \diagdown & & \\ & \omega & \\ & & \diagdown \end{bmatrix} \phi^{-1} \qquad \ldots(3)$$

The matrix C thus computed is a full symmetric matrix, but the diagonal elements were dominant, being in most rows at least two times the magnitude of the largest off-diagonal element which was largely negative. For our analyses we diagonalized C by simply discarding all the off-diagonal elements.

NUMERICAL STABILITY AND COMPUTER PROGRAMING

Normal modes and frequencies were computed for both the cracked and uncracked dams. The frequencies for the uncracked dam compared very well with those found by Saini[7].

To have numerical stability in the step-by-step integration the step size must be a fraction of the smallest period. The time step used for the uncracked dam was 0·002 sec (about 50 percent of the smallest period). It gave stable results. Doubling this step size, however, yielded unstable results. For the cracked dam the step size was reduced to 0.0005 sec to give satisfactory results. This was due to the fact that the cracked dam had two additional degrees of freedom of motion giving rise to a smaller "smallest period" than the uncracked dam.

The effect of step size on the computed accelerations was much more pronounced than the effect on computed displacements. Results were computed for the cracked dam using time steps of 0.001 sec and 0.0005 sec. The differences in the computed dis-placements were very small (within a few percent), but the differences in the computed accelerations were as high as 100 percent or more. Since the maximum stresses are the critical response parameters, and stresses are functions of displacements only, one may tolerate inaccuracies in computed accelerations as long as displacement accuracy is not impaired. Hence a time step of .0005 sec was considered acceptable. Similar observations were reported by Tahbildar and Tottenham[9].

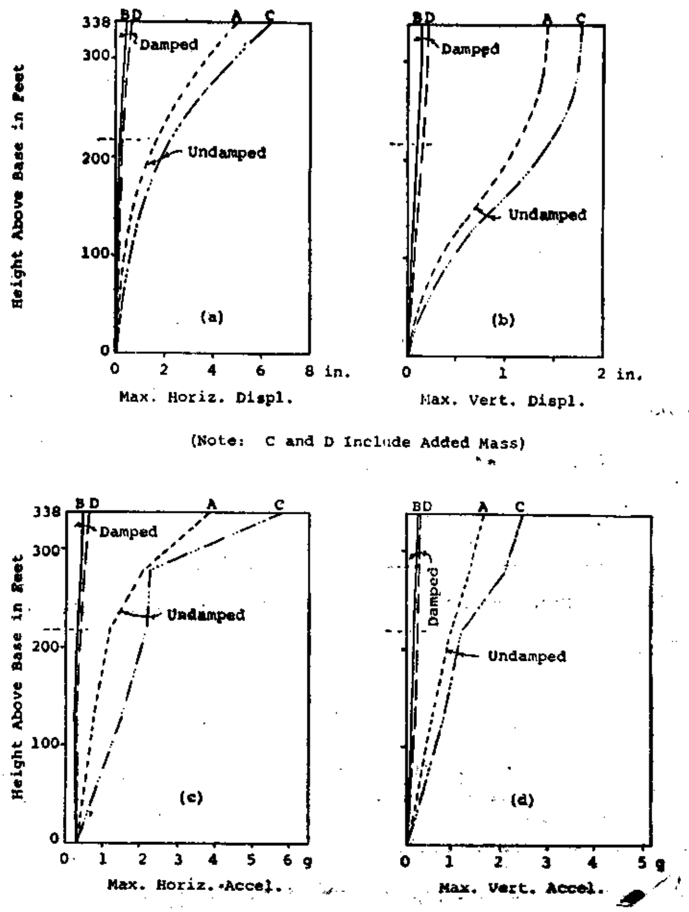
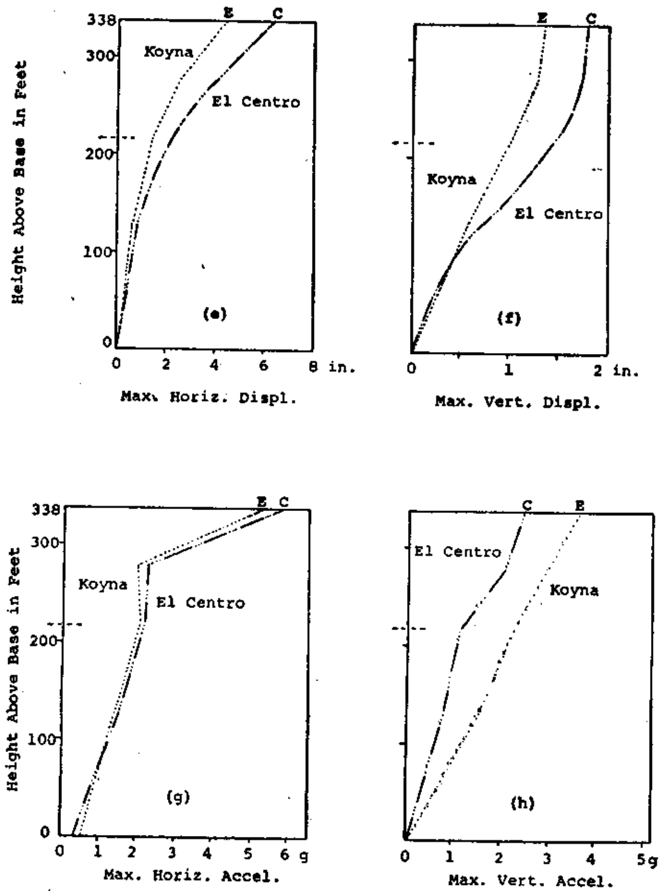


Fig. 3. Max. Displ. and Max. Accel. of Upstream Face for Various Cases of Dam.

The presence of the horizontal crack in the dam seems to cause a large magnification in the computed accelerations. Since nodal points immediately above and below the crack may have different vertical displacements, velocities, and accelerations in the cracked state but not in the uncracked state, the sudden transition from one state to the other leads to large computed accelerations at the transition times. This phenomenon has negligible effect on the displacement response.



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It is necessary to use double precision arithmetic in the calculations. The computation time needed, on an IBM 360–67 computer, for each time step of integration was approximately 0.2 sec for the cracked dam. It took 40 minutes to analyze the dam for the first 7.12 sec of the El Centro (NS) earthquake. Some 14,000 integrations were performed with a maximum time step of 0.0005 sec.

COMPUTED RESULTS FOR KOYNA DAM

The tallest monolith (monolith No. 18 [11]) of Koyna Dam, as shown in Fig. 1, was chosen as the model to be studied. The rubble concrete gravity dam was assumed to be homogeneous and isotropic with Young's modulus of 3,000,000 psi and Poisson's ratio of 0.2. The dynamic response to the two earthquakes were considered, namely, El Centro, May 18, 1940. NS component, and Koyna, December 11, 1967, transverse component.

DISPLACEMENTS

The maximum relative displacements and the maximum absolute accelerations of the upstream face of the dam are shown plotted in Fig. 3 for various parameters which are shown tabulated and designated below.

Case	Type	Added Mass of Water	Damping	Earthquake
A	Uncracked	No	No	El Centro
B	Uncracked	No	Yes	El Centro
C	Uncracked	Yes	No	El Centro
D	Uncracked	Yes	Yes	El Centro
E	Uncracked	Yes	No	Koyna
F	Cracked	No	No	El Centro

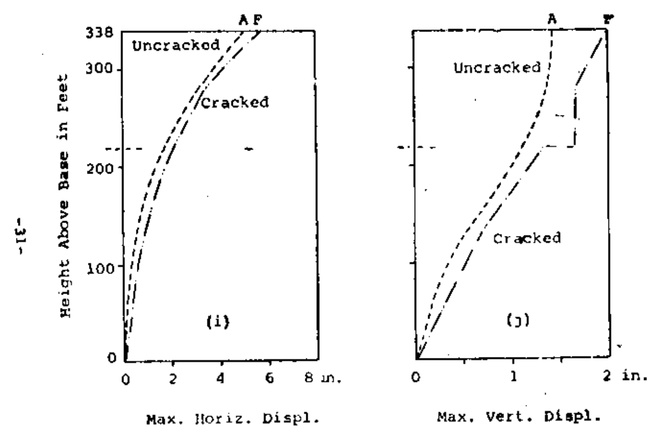

Fig. 3. Max. Displ. and Max, Accel. of Upstream Face for Various Cases of Dam.

As would be expected, the presence of added mass due to the water in the reservoir, Case "C" in Fig. 3, caused greater maximum displacements in the dam. The upstream top point of the dam deflected 27 percent more horizontally and 23 percent more vertically than the corresponding point in Case "A" which had no added mass. On the other hand for the same point the accelerations in Case "C" were 52 percent greater than those of Case "A". The presence of damping, Cases "D" and "B" in Fig. 3, even though reduced greatly the overall response, did not alter substantially the relationship mentioned above due to added mass.

Maximum displacements caused by Koyna earthquake Case "E" in Fig. 3 were much smaller than the corresponding displacements produced by El Centro earthquake Case "C". At the upstream top point, a reduction of 30 percent in the horizontal and 23 percent in the vertical displacements were noticed. The acceleratian response results were quite different and may be attributable to integration step size problem discussed earlier.

The presence of damping, Cases "B" and "D" Fig. 3, (equivalent to 5 percent of critical damping in each normal mode) proved very effective. The maximum displacements were reduced from the corresponding undamped case (Cases "A" and "C" in Fig. 3) by approximately 90 percent. Also this effectiveness may have been in part magnified due to the omission of the off-diagonal terms of the damping matrix, since off-diagonal terms were mostly negative as mentioned earlier in this paper.

For the cracked dam. Case "F" in Fig. 3, the maximum displacement of the upstream top point was 15.2 percent larger horizontally and 38.6 percent larger vertically than for the uncracked dam, Case "A" in Fig. 3. On the upstream surface of the dam, the maximum displacement of the upper point at the crack was 0.33" more vertically than the lower point. The value of the former was 48.8 percent greater, and the latter 19 percent greater than the displacement of the corresponding point in the uncracked dam, Case "A".

In all the cases considered the maximum horizontal displacement of the upstream face resembled the first mode shape of a cantilever column.

STRESSES

Unlike the maximum displacements, the stresses were not obtained at every time step. They were calculated and recorded only at accelerogram points.

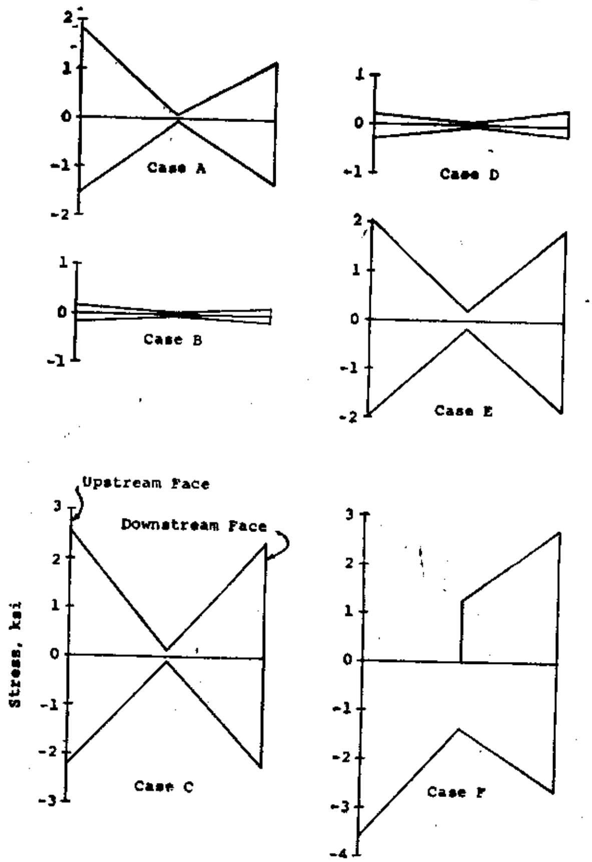

Fig. 4. Maximum and Minimum Vertical Stresses in Dam Across Crack Level (El. 218 Ft.) Due to Earthquake.

The maximum and minimum stresses then refer to the extreme values of the recorded stresses. The absolute maximum and absolute minimum values of the stresses may somewhat vary from the ones presented and discussed in this article. One would expect them not to be substantially different, however.

The maximum and minimum values of the vertical stress for El. 218 ft (crack level) are shown plotted in Fig. 4 for all the six cases considered. The variation in the stresses between the different cases is indeed very similar to the variation in displacements discussed earlier. The only exception is found in the Koyna earthquake Case "E" in Fig. 4 where one would have expected smaller stresses than the stresses in Case "A", since Case "A" had greater maximum relative displacements (see Fig. 3) than Case "E". The increase in the stress in Case "E" over that of Case "A" however may not necessarily reflect a true picture. Absolute extreme stresses were not calculated at every step; hence a final conclusion about this apparent difference of behavior cannot be reached without further study.

The cracked dam, Case "F" in Fig. 4, exhibited the greatest compression and the greatest tension across the horizontal section at crack level. In actual design these figures have to be combined with the stresses caused by hydrostatic forces, as well as the weight of the dam itself. There has been no attempt made to evaluate the stress concentration due to the presence of the crack. A refined mesh would be needed for accomplishing this purpose and is beyond the scope of the present study.

DISPLACEMENT AND ACCELERATION TIME PLOTS

Time history plots of the displacements and acceleration of the upstream face of the uncracked dam for various cases are shown plotted in Figs. 5 through 8. The plots were constructed using straight line segments between the calculated points. It is noticed that the maximum values of the displacements occurred at about 5 secs after the start of the earthquakes. Thus using 7.15 secs of the accelerograms for the earthquake considered seems to be justified. The displacement plots show the dominance of the fundamental mode of vibration in the undamped cases.

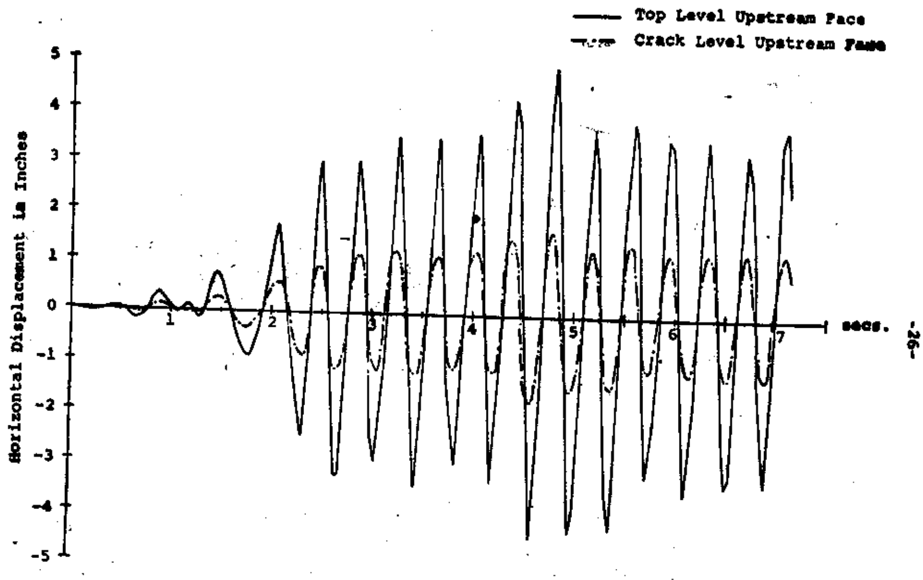

Fig. 5. Horizontal Displacement History for Dam, Case A.
(Plots constructed using straight line segments between points)

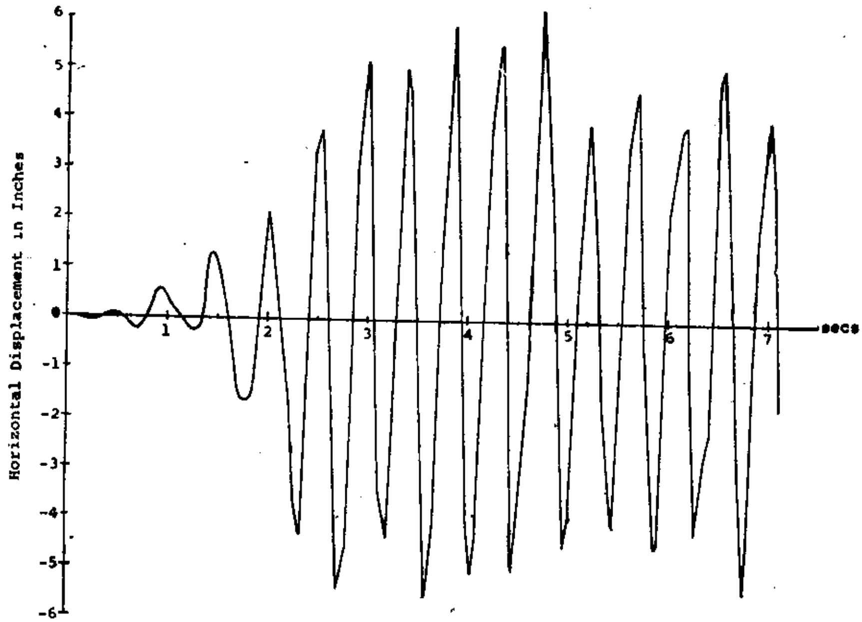

Fig. 6. Horizontal Displacement History for Top Level (Upstream Face) Dam, Case C. (Plots constructed using straight line segments between points)

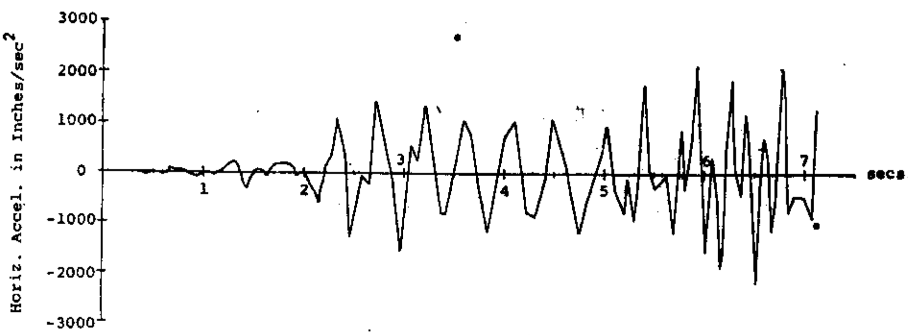

Fig. 7. Horizontal Acceleration History for Top Level (Upstream Face) Dam, Case C. (Plots constructed using straight line segments between points)

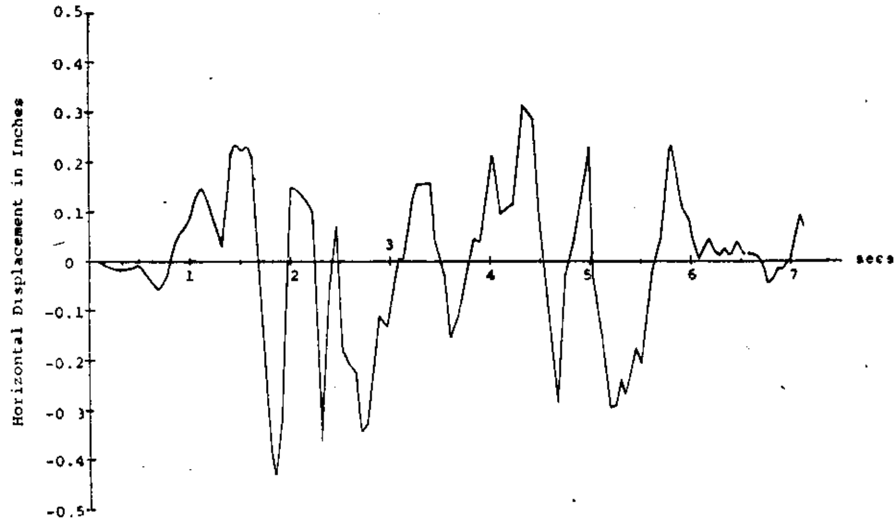

Fig. 8, Horizontal Displacement History for Top Level (Upstream Face) Dam, Case B. (Plots constructed using straight line segments between points)

Figures 9 and 10 show displacement plots for the cracked dam, Case "F". In general the observations made above for the uncracked dam are also applicable here.

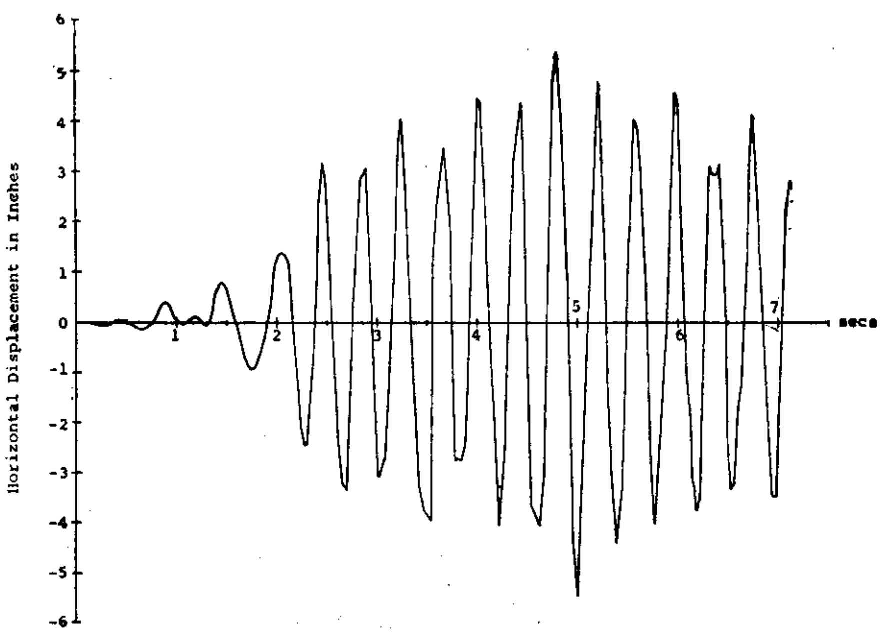

Fig. 9. Horizontal Displacement History for Top Level (Upstream Face) Cracked Dam, Case F. (Plots constructed using straight line segments between points)

Figure 10 shows the vertical separation between the two surfaces of the crack at the upstream face of the dam. Note that the separation between the two surfaces exceeded a number of times the half inch mark.

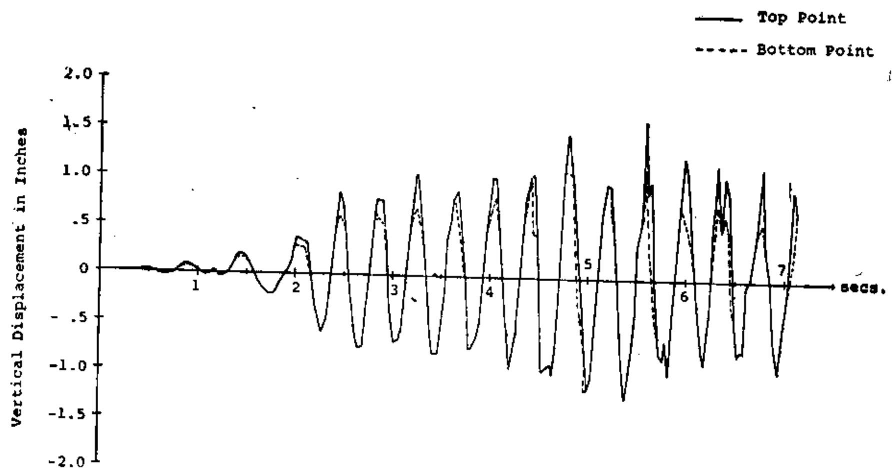

Fig. 10. Vertical Displacement History at Crack Level (Upstream Face) Cracked Dam, Case F. (Plots constructed using straight line segments between points)

CONCLUSIONS AND RECOMMENDATIONS

The present study has demonstrated that it is indeed possible with very few isoparametric elements, two in this case, to form a realistic model of a dam for dynamic studies, and obtain good results within reasonable computer time.

Non-linearities due to geometry, like horizontal cracks, can adequately be handled with such elements. The application of linkage elements would allow also hydrostatic pressure and friction effects in the cracks to be included. This was not done in the present study.

The use of an approximate diagonal mass matrix seems reasonable. However, further study is needed to find the best way to distribute the mass of an element among its nodal points. Consistent mass matrix should also be considered.

The presence of a partial crack substantially increased the displacement response and caused greater stresses in tension as well as in compression at the level of the crack.

Added mass due to water in the reservoir caused larger displacements.

In the direct numerical integration method, the acceleration response, unlike the displacement response, is found to be very sensitive to step size. For good displacement results the step size must not exceed one half of the smallest period of the dam. The greater the number of degrees of freedom of a dam, the smaller is the highest period. Thus it is advantageous to discretize the dam with fewer elements.

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