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SEISMIC ANALYSIS OF TYPICAL MULTISTOREYED FRAMES

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INTRODUCTION

The structures that are constructed on geologically unstable region where severe or moderately severe earthquakes are experienced and are likely to reoccur, should be designed considering the dynamic behaviour against possible earthquake effects apart from all other conventional design considerations. In view of the heavy construction programme in India, the Indian standard code of practice for Earthquake Resistant Design, based on seismic data obtained from the studies of past Indian earthquakes was first published in 1962. Through revisions the present revised copy has been made available as I.S: 1893–1975, Criteria For Earthquake Resistant Design of Structures (Third Revision).

A Modal Analysis based on structural dynamics is presented here with few reasonable assumptions suggested by Blume[2] in computing stiffness matrix. Matrix Iteration[3,4] along with Zooing Procedure[4,5] can be conveniently used to solve for the vibration problems. The stiffness matrix as computed here deals with the stiffness of column elements for a transverse displacement only, ignoring joint rotation. This enables a tri-diagonalisation nature of the stiffness matrix which can be conveniently inverted to form the eigen value equation of the dynamic problem as shown subsequently. Furthermore, eigen values representing frequencies of vibration and eigen vectors representing model shapes of the structure should be obtained for at least three fundamental modes as suggested by Clough[6], so that although the effects of first mode is evidently dominant, some effects of subsequent modes could be superimposed to get a more realistic value against a total vibrational effect due to earthquake.

MODAL ANALYSIS

The well known dynamic equation applicable to a structural element in vibration is of the form—

$$\left[S^{-1}\right]\left[\diagdown M \diagdown\right]\left\{\delta\right\}=\frac{1}{\omega^2}\left\{\delta\right\} \qquad \ldots(1)$$

where,

$[S]$ = stiffness matrix of the structural element

$\left[\diagdown M \diagdown\right]$ = lumped mass matrix of the structural element

$\{\delta\}$ = nodal displacement vector.

The same equation may be applied to a multistoreyed building frame as a whole, by considering lumped mass idealisation of the structural system, which is represented by a vertical cantilever with masses lumped at respective storey level. Mass matrix is formed by placing the masses diagonally and stiffness matrix is computed assuming 'closed-coupled' system defined by Blume[2]. Evidently, Eq. (1) is an Eigen Value problem and the solution may be obtained by Matrix Iteration[3,4] and Zooing Procedure[4,5].

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Zooing procedure is the convenient method of removing roots from the original equation one after the other.

For the explanation of the assumptions made in computation of storey stiffness following isolated hypothetical freedoms as defined by Blume[2] are discussed below:

1. **Shear Deformation:** Shear Deformation is the horizontal deformation caused by an external force applied to a storey, the rest of the storeyes being prevented from moving and with the assumptions that there is no joint rotation and vertical deformation of vertical elements. Shear Deformation is obtained by relative stiffness of all structural elements, considering Shear Deformation and bending of columns, using shear areas and moment of inertia of individual columns. The system was termed by Blume[2] as 'closed-Coupled' system.

2. **Joint Rotation Deformation:** Joint Rotation Deformation is the horizontal deformation in a frame type building obtained by relaxing rotational restraint condition imposed in shear deformation.

3. **Over-all Flexture Deformation:** Over-all Flexture Deformation is the horizontal deformation obtained by relaxing the infinitely large axial rigidity condition imposed in Shear Deformation.

4. **Base Deformation:** Base Deformation is the horizontal or X-axis deformation by rocking of the foundation about Z-axis (the other horizontal axis) through the base of the foundation.

Traditional multistoreyed building frames are normally of short span and have rigid floor system. This practically nullifies the effect of Joint Rotation Deformation. It was shown in Reference (2) that effect of over-all flexture on natural time period of structures is negligible. The foundation of traditional buildings are assumed to be sufficiently rigid. So shear deformation is the only significant freedom at each storey which is considered for computing storey stiffness.

Storey stiffness for jth storey of a frame where Shear Deformation is the only significant freedom was given by Blume[2] as—

$$S_j = \frac{12E}{h_j} \sum_{K=1}^{all} \left(\frac{1}{\phi_K}\right) \qquad \ldots(2)$$

where,

h_j=height of the jth storey,

$\phi_K = \frac{h_j^2}{I_K} + \frac{30}{A_{VK}}$ (if units are in Kips inches)

I_K=moment of inertia of kth vertical element

A_{VK}=effective shear area of kth vertical element

E=Young's modulus of the structural element.

In computing storey stiffness for the case studies presented next, the shear area term in ϕ_X is neglected since contribution of this term to storey stiffness is negligibly small. The over-all stiffness matrix of a lumped mass idealised structure for a 'Closed-Coupled' system is given by the following tridiagonal matrix.

$$[S]=\begin{bmatrix} (S_1+S_2 & -S_2 & 0\ldots\ldots & 0 & 0 & 0 \\ -S_2 & (S_2+S_3) & -S_3\ldots\ldots & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0\ldots\ldots & -S_{n-1} & (S_{n-1}+S_n) & -S_n \\ 0 & 0 & 0\ldots\ldots & 0 & -S_n & S_n \end{bmatrix} \quad \ldots(3)$$

Forming Eq. (1) by over-all stiffness matrix $[S]$ and mass matrix $\left[\diagdown M \diagdown\right]$, obtained by lumped mass idealisation of a structural system, and solving the equation by Iterative and Zooing Procedure, natural frequency of vibration ω_i and consequently natural time period of vibration

$$T_i=\frac{2\pi}{\omega_i} \quad \ldots(4)$$

for ith mode may be obtained.

Clough[6] expressed the Base Shear B_i for ith mode in terms of Spectral Velocity V_i for ith mode as

$$B_i=\frac{W_i}{g}\cdot\frac{2\pi}{T_i}.V_i \quad \ldots(5)$$

where W_i=effective weight for ith mode.

Spectral Velocity was defined by Clough[6] as the maximum velocity produced in the structure by a particular ground motion and is obtained by a curve, known as velocity spectrum curve, which shows maximum velocity produced by this particular ground motion for a complete spectrum of period of vibration of the structure. Obviously, velocity spectrum is different for different earthquakes and hence for practical use of the curve average characteristics of velocity spectra was essential. These Characteristics were evaluated by G.W. Housner[6] and shown here in Fig. 1 (Reference 4, Fig. 6.4).

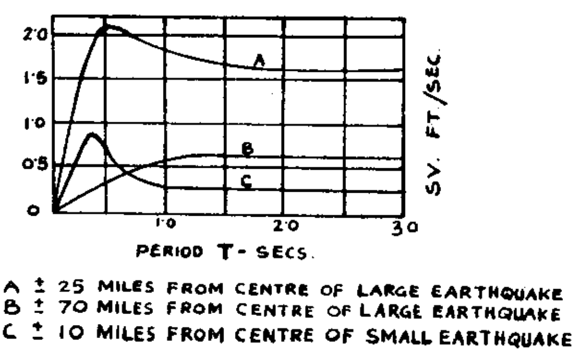


Fig. 1

Effective weight W_i in Eq. (5) was defined by Clough[6] as

$$W_i = \frac{\{D_{ki} M_k\}^2}{\Sigma D_{ki} M_k} \cdot g \qquad \ldots(6)$$

where,

D_{ki} = deflection at kth storey level for ith mode,

M_k = mass at kth storey level

g = acceleration due to gravity

Clough[6] has adopted a distribution Coefficient as

$$\frac{D_{ki} M_k}{\Sigma D_{ki} M_k} \qquad \ldots(7)$$

and Base Shear is distributed as Earthquake Force as

$$F_{ki} = B_i \left(\frac{D_{ki} M_k}{\Sigma D_{ki} M_k} \right) \qquad \ldots(8)$$

where,

F_{ki} = earthquake loading at kth storey level for ith mode.

To obtain the total force developed throughout the height of building clough[6] suggested to add 50% of second mode and third mode storey shears to the corresponding first mode storey shears to take into account the effect of higher modes. It is absolutely rational rather than increasing the fundamental mode response by a constant factor because effect at a particular storey for different modes are different in nature.

CASE STUDY

Two case studies are presented here based on Case B of Housner's Curve (Fig. 1), which corresponds to a distance of ±70 miles from the centre of large earthquake. In analysis as per Seismic Coefficient Method of I.S. Code[1], basic horizontal seismic coefficient (α_0) values in both the case studies are adopted in such a way, so that the same base shear values are obtained in both the methods of analysis to form the basis of comparison and value of β, a coefficient depending on soil-foundation system is taken as 1.2 and the value of I, a coefficient depending on importance of the structure is taken as 1.

Case Study I : A Five Storeyed Symmetrical Frame.

The geometry of the frame is shown in Fig, 2.

Relevant data are as follows :

Frames are spaced at 4M C/C

Relevant Moment of Inertia of Columns, $I_k = 8372.2$ cm^4

Intensity of dead load = 925 kg/m^2

Intensity of live load = 300 kg/m^2

Total intensity of loading = 1225 kg/m^2

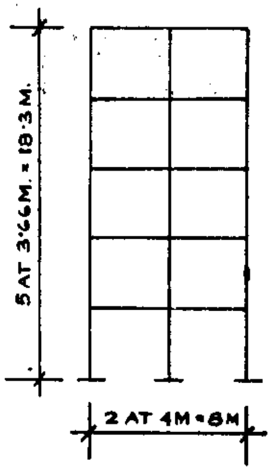


Fig. 2

Load per floor, W=32000 kg. [Appropriate reduction (25%) of live load as per I.S. Code[1] has been considered].

Young's Modulus of Steel, $E_s = 2 \times 10^6$ kg/cm^2

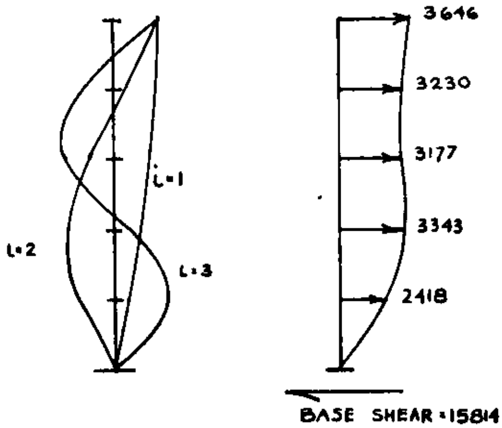
The results obtained through modal analysis as suggested earlier are given in Table 1.

TABLE 1

EIGEN VECTOR	$i=1$	$i=2$	$i=3$
D_{5i}	1	1	1
D_{4i}	0.919	0.310	−0.715
D_{3i}	0.763	−0.594	−1.204
D_{2i}	0.546	−1.088	0.373
D_{1i}	0.285	−0.831	1.31
ω_i IN RAD. PER SEC	5.53	16.13	25.43
T_i IN SEC.	1.14	0 39	0.25
W_i IN KGS.	140740	13935	3891
V_i IN CM./SEC.	18.29	9.14	6.1

A simple computer program[4] was used incorporating Matrix Iteration and Zooing Procedure as explained earlier for the three consecutive modes.

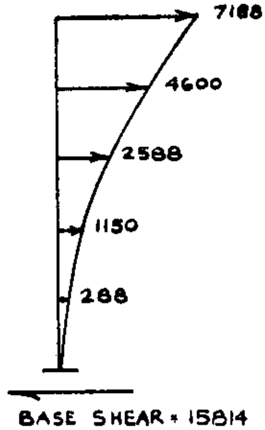
The result of the Modal Analysis is presented in Fig. 3. The result of the analysis as per the I.S. Code[1] on the same frame is shown in Fig. 4. In method as per I.S. Code[1], Basic Horizontal Seismic Coefficient (α_0) value is taken as 0.1 for the purpose of making the base shear value same in both the methods. Value of β is taken as 1.2 and value of I is taken as 1.



(a) Mode shape

(b) Earthquake loading in kgs. as per modal analysis

Fig. 3



Earthquake loading in kgs as per IS: 1893–1975

Fig. 4

Case Study 2: A Ten Storeyed Symmetrical Steel Frame.

The geometry of the frame is shown in Fig. 5.

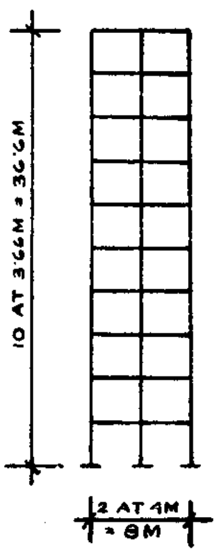


Fig. 5

Relevant data are as follow:

Frames are spaced at $4M$ C/C.

Relevant Moment of Inertia of Columns, $I_k = 16744.4$ Cm^4

Intensity of dead load=925 kg/m^2

Intensity of live load=300 kg/m^2

Total intensity of loading=1225 kg/m^2

Load per floor, $W=32000$ kg [Appropriate reduction (25%) of live load as per I.S. Code[1] has been considered.]

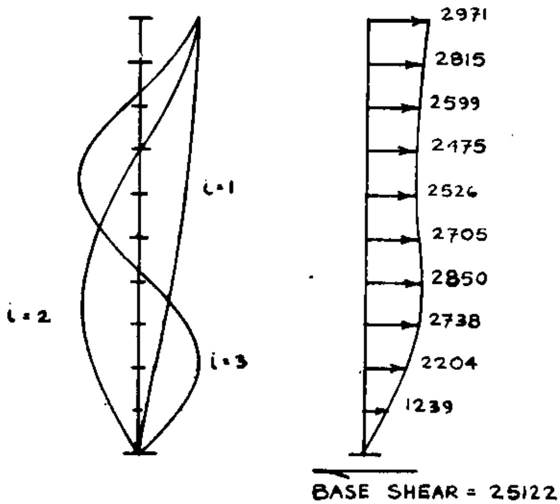
Young's Modulus of Steel, $E_s = 2\times 10^6$ kg/cm^2

The results of the frame obtained from the Modal Analysis are given in Table 2:

TABLE 2

EIGEN VECTOR	$i=1$	$i=2$	$i=3$
D_{10i}	1	1	1
D_{9i}	0.978	0.802	0.446
D_{8i}	0.933	0.445	−0.317
D_{7i}	0.868	0	−0.930
D_{6i}	0.784	−0.445	−1.047
D_{5i}	0.682	−0.802	−0.605
D_{4i}	0.565	−1	0.160
D_{3i}	0.435	−1	0.840
D_{2i}	0.295	−0.802	1.071
D_{1i}	0.149	−0.445	0.731
ω_i In RAD. PER SEC	4.1	12.2	20.06
T_i IN SEC	1.53	0.51	0.31
W_i IN KGS	271378	29254	9901
V_i IN CM/SEC	19.81	10.67	6.4

The result of Modal Analysis is shown in Fig. 6. The result as per I.S. Code[1] on the same frame is shown in Fig. 7. In the method of analysis as per I.S. Code[1], the value of Basic Horizontal Seismic Coefficient (α_0) is taken as 0.12, to make the base shear value same as in Modal Analysis. The values of β and I are taken same as in case study 1.



(a) (b)

Fig. 6 Mode shape — Earthquake loading in kgs as per model analysis

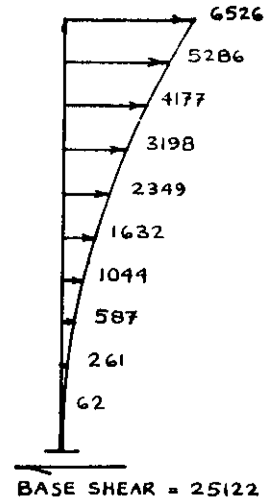


Fig. 7 Earthquake loading in kgs as per I,S.: 1893–1975

CONCLUDING REMARKS

Comparing the results obtained by the Modal Analysis presented here with the results obtained by the method specified in I.S.: 1893–1975 (Reference Fig. 3, Fig. 4, and Fig. 6, Fig. 7) it appears that the nature of loading in the former method approximates to a more uniform type than the latter, where the loading is towards triangular in nature. The reason of this deviation in nature of loading may be due to the superimposition of the higher mode effects on that in fundamental mode in the Modal Analysis presented here. It is apparent from the results obtained by the two methods, the response at mid height of the structures are fairly in agreement, whereas, in comparison to response in Modal Analysis, earthquake loading at higher storeys are much higher and at lower storeys are lower in methods as per Indian Standard Code of Practice.[1]

The Modal Analysis deliberated here provides a relatively simple and sufficiently accurate method of determining stiffness, period of vibration, mode shapes and earthquake loading of a structural frame with the only significant freedom, that is Shear Deformation, and may be conveniently adopted for seismic analysis of multistoreyed frame structure, as it can be easily computerised.

The advantage of rotational restraint assumption made at each mode in computing mass matrix is similar to that in the computation of stiffness matrix. This assumption reduces the order of mass matrix which is necessarily made equal to the order of stiffness matrix. Solution of the Eigen Values and Eigen Vectors for a large degree of freedom system can be obtained by simple computer program presented in Reference (4).

NOTATIONS

A_{vk} =Effective shear area of kth vertical element

B_i =Base shear for ith mode

D_{ki} =Deflection at kth storey level for ith mode

E =Young's modulus of structural element

E_s =Young's modulus of steel

F_{ki} =Earthquake loading at kth storey level for ith mode

g =Acceleration due to gravity

h_j =Height of the jth storey

I_k =Moment of inertia of kth vertical element

$\left[\nwarrow M \searrow\right]$ =Lumped mass matrix

S_j =Storey stiffness for jth storey

$[S]$ =Stiffness matrix

T_i =Time period of vibration for ith mode

V_i =Spectral velocity for ith mode

W =Load per floor

W_i =Effective weight for ith mode

α_0 =Basic horizontal seismic coefficient

δ =Amplitude of vibration

$\{\delta\}$ =Nodal displacement vector

ω_i =Natural frequency of vibration for ith mode

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