Paper No. 136, Bulletin I. E. S. T., Vol. 10, No. 3, pp. 113-123, September 1973

STATIC AND DYNAMIC ACTIVE EARTH PRESSURES BEHIND RETAINING WALLS

P. NANDAKUMARAN* AND V. H. JOSHI*

INTRODUCTION

There are many methods for estimating the earth pressure behind an earth retaining wall. Almost all of them are helpful in determining the magnitude of the earth-pressure force. They do not normally speak about the distribution of earth pressure behind the wall, or the point of action of the resultant force. As such, it is tacitly assumed that the earth pressure has a pattern of hydrostatic pressure distribution and that the resultant acts at one third height from the base of the wall; though the experimental results are not always concurrent with this assumption. In case of the dynamic increment of earth pressure, the I, S. Code arbitrarily fixes the point of its action at two third height from the base of the wall and experimental evidence to prove the same is very much wanting.

It would be rather easy to arrive at the point of application of earth pressure as well as that of dynamic increment, if it is possible to assess the distribution of the normal reaction of the soil along the rupture plane. In the proposed analysis, the normal reaction is assumed to vary linearly with the depth of the soil above. The results of a limited number of graphical solutions have shown that this approach can successfully predict the point of application of static earth pressure and the dynamic increments, which reasonably agree with the experimental data available.

ASSUMPTIONS MADE IN THE PROPOSED METHOD OF APPROACH

1. The back of retaining wall is rough.
2. The rupture surface behind the retaining wall is a plane one.
3. The wall has yielded sufficiently, so that the state of active earth pressure is reached.
4. The failure wedge behind the wall is the same in both static as well as the dynamic conditions.
5. The soil is a cohesionless material.

All the above assumptions excepting the last are the well known assumptions made by Coulomb in his classical theory of earth pressures. Though the failure surface is not truly plane surface, yet, for the purpose of simplicity it is assumed to be plane one; this can be easily obtained by using the earth pressure tables prepared by Jumikis (4).

Since all the retaining walls are usually designed for active earth pressures only, it is reasonable to assume that the wall has yielded sufficiently to bring about active earth pressure conditions behind it. Once the rupture surface is developed, the occurrence of earthquakes tends to produce further displacements of the same failure wedge rather than to cause a fresh one. This has been observed in the studies of Prakash and Nandakumaran (11). As such it is reasonable to assume that the failure wedge under dynamic conditions is the same as that in static conditions.

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* Reader and Lecturer respectively, School of Research and Training in Earthquake Engineering, University of Roorkee, Roorkee, U. P.

FORCES ACTING ON THE FAILURE WEDGE

(i) Static conditions.

Under the static conditions, three forces keep the failure wedge in static equilibrium (Fig. 2). One is the weight of the soil in failure wedge which is known in magnitude, direction, and the point of application. The other is normal reaction of the soil along the rupture plane, whose direction is known. The normal reaction at any point on the rupture plane, is a function of the vertical stress at that point, which in turn is a function of the depth of the soil column above that point. Thus, it is quite reasonable to assume that the normal reaction along the rupture plane varies linearly with the depth of soil wedge above which determines the point of application of the normal reaction. The third force is that due to earth pressure which is known in direction. Using this data and employing the principles of static equilibrium, the point of application of the earth pressure force can be obtained easily.

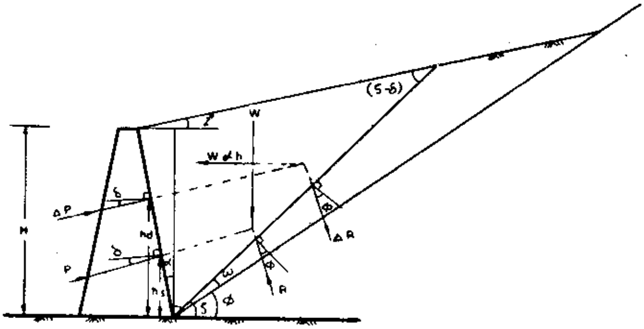

Fig. 1. Various forces keeping the failure wedge in equilibrium under static and dynamic condition

(ii) Dynamic case.

In this case, in addition to the earth pressure force known in direction and the weight of the soil in the failure wedge known in magnitude, direction and point of action, the inertia force due to the soil wedge also acts, which is also known in magnitude, direction and point of action (Fig. 1 and Fig. 2).

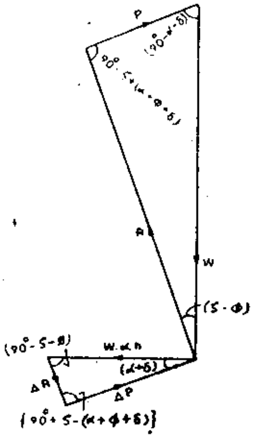

Fig. 2. Force diagram for static and dynamic cases

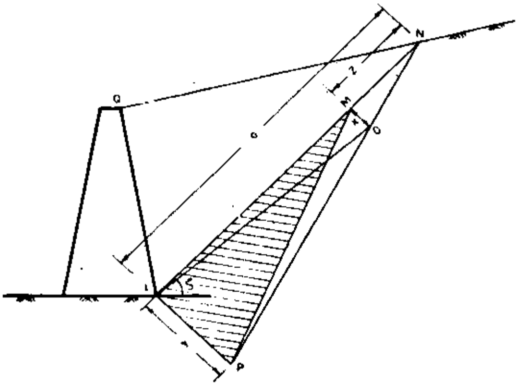

Fig. 3. Distribution of pressure due to soil reaction and its dynamic decrement along rupture plane

In most cases, the retaining walls tend to fail by rotation about the base because the wall is well keyed into the ground for translation movements, and also because it is subjected to large overturning moments. As such, it can be taken to be so under dynamic conditions also. So, when the failure wedge tends to rotate about the base under dynamic case, it leads to no lateral displacements at the base of the wall, whereas the tendency of the wedge moving away from the soil behind it will increase *linearly* with the increase in height above the base of the wall, which leads to a definite decrease in the normal reaction along the rupture plane. Since tendency for the outward movement is maximum at the top and nil at the base, it is assumed that the dynamic decrement in normal reaction varies linearly, with the maximum value at the ground surface and zero at the base of the wall.

But this assumption will lead to negative soil reaction near top portion of the rupture since the static soil reaction is assumed to start with zero value at the ground level. So, it is reasonable to modify this assumption to state that the decrement in soil reaction at any point along the rupture plane does not exceed the value of the soil reaction under static condition at that point. This is quite reasonable for the cohesionless soils, for which the analysis is proposed.

ANALYTICAL PROCEDURE FOR OBTAINING THE POINT OF APPLICATION OF SOIL REACTION

By employing the principles of static equilibrium i.e. $\Sigma H=0$, $\Sigma V=0$ and $\Sigma M=0$; the magnitude and point of action of earth pressure force can be obtained if the weight of the soil wedge and the soil reaction are known completely. Since the reaction is assumed to vary linearly the resultant of it acts at one third height from the base of the wall in static conditions.

Under the dynamic conditions the principle of superposition is assumed to hold good i.e. in addition to the forces in the static condition the additional forces that are produced due to dynamic condition should also satisfy the principles of static equilibrium. The resultant dynamic earth pressure force will be the vectorial sum of the static earth pressure force and the dynamic increment.

In Fig. 3, $\triangle$LMNOP represents distribution of static soil reaction: and $\triangle$LMNO ($\equiv\triangle$PMNO) that of the dynamic decrement in soil reaction. The resultant soil reaction is represented by $\triangle LMP$. Let y be the intensity of static soil reaction at base of the wall. Z the distance along the rupture plane measured from the top end, at which the dynamic decrement of soil reaction is maximum ($=x$;). If the value of Z is known, it is quite easy to arrive at the point of application of the resultant soil reaction under dynamic conditions. With the notations as listed at the end of the paper and as shown in the figure 3, we have,

Soil reaction under static conditions $=R$

$$=\frac{p\sin(90^\circ-\alpha+\delta)}{\sin(\zeta-\Phi)}$$

$$=\tfrac{1}{2}.\gamma.H^2.\frac{\cos(i-\alpha)}{\cos^2\alpha}.\frac{\cos(\zeta-\alpha)\cos(\zeta-\Phi)}{\sin(\zeta-i)\sin\{90^\circ+\zeta-(\alpha+\Phi+\delta)\}}\frac{\sin(90^\circ-\alpha+\delta)}{\sin(90^\circ-\Phi)} \tag{1}$$

Weight of the soil wedge $=W$

$$=\tfrac{1}{2}.\gamma.H^2\frac{\cos(\zeta-\alpha)}{\cos^2\alpha}\frac{\cos(i-\alpha)}{\cos(\zeta-i)} \tag{2}$$

Dynamic decrement in soil reaction $=\triangle R$

$$=\frac{W\alpha h\sin(\alpha+\delta)}{\sin(90^\circ+\zeta-(\alpha+\Phi+\delta))\}}$$

$$=\alpha_h\frac{\gamma h^2}{2}\frac{\cos(\zeta-\alpha)}{\cos^2\alpha}\frac{\cos(i-\alpha)}{\sin(\zeta-i)}\frac{\sin(\alpha+\delta)}{\sin\{90^\circ+\zeta-(\alpha+\Phi+\delta)} \tag{3}$$

Since the soil reaction is assumed to vary linearly with depth, we have

$$R=\tfrac{1}{2}.y.a. \tag{4}$$

wheae $a=$length of the rupture plane.

Let "x" be the intensity of the soil reaction at the distance at the distance "Z" from the top end of the rupture plane, form which we get,

Dynamic decrement in soil reaction $=\triangle R$

$$=\triangle MNP=\triangle LNO=a.x/2 \tag{5}$$

and

$$Z=(x/y).a \tag{6}$$

From Fig. 2; by applying sine rule to the force polygon, we get

$$\frac{(H/\cos\alpha)}{\sin(\zeta-i)}=\frac{a}{\sin(90+i-\alpha)}$$

$$\therefore\quad a=\frac{H\sin(90+i-\alpha)}{\cos\alpha\sin(\zeta-i)} \tag{7}$$

From equation (5) and (4) we get

$$y=\frac{2R}{a}\text{ and }x=\frac{2\triangle R}{a}$$

On substituting the values of y and x in Eq. (6) we get

$$Z=\frac{\triangle R}{R}.a \tag{8}$$

$$=H\alpha_h\frac{\sin(\alpha+\delta)\sin(\zeta-\Phi)\sin(90+\delta-\alpha)}{\sin(90-\alpha-\delta)\sin(\zeta-\delta)\cos\alpha\cos(\zeta-\Phi)} \tag{9}$$

Thus, knowing the value of Z, the point of application of resultant soil reaction under dynamic condition and hence that of the dynamic earth pressure can be easily obtained.

GRAPHICAL METHOD OF ANALYSIS

Since the expressions involved in the computation of earth pressure are quite lengthy and complicated for analytical solution, it is easier to obtain graphical solutions. The graphical approach can be split into two parts, one for determining the static earth pressure and the other for the dynamic increment ($\triangle P$). Equation (8) comes out to be handy in calculating the value of Z.

EFFECT OF VARIOUS PARAMETERS ON THE POINT OF APPLICATION OF THE EARTH PRESSURE FORCE.

In most of the conventional earth pressure theories the point of application of the earth pressure force is assumed to be at a height (h_s) of one third the height of the wall above the base. But the proposed approach shows that h_s is a function of angle of wall friction (δ), angle of surcharge (i), angle of wall back with vertical (α) and the angle of shearing assistance of the soil (Φ).

Fig 4 shows that h_s decreases with increasing value of angle of wall friction δ. But for a smooth wall ($\delta=0$), h_s is equal to one third the height of the wall. Fig. 5 shows that h_s increases with increasing values of angle of surcharge (i), the variation being sharper for larger values of i. In Fig. 6 it can be seen that h_s increases again with increasing values of angle of wall back with vertical, for the walls leaning away from the fill material. But in this case, the variation is more sharp for smaller values of α.

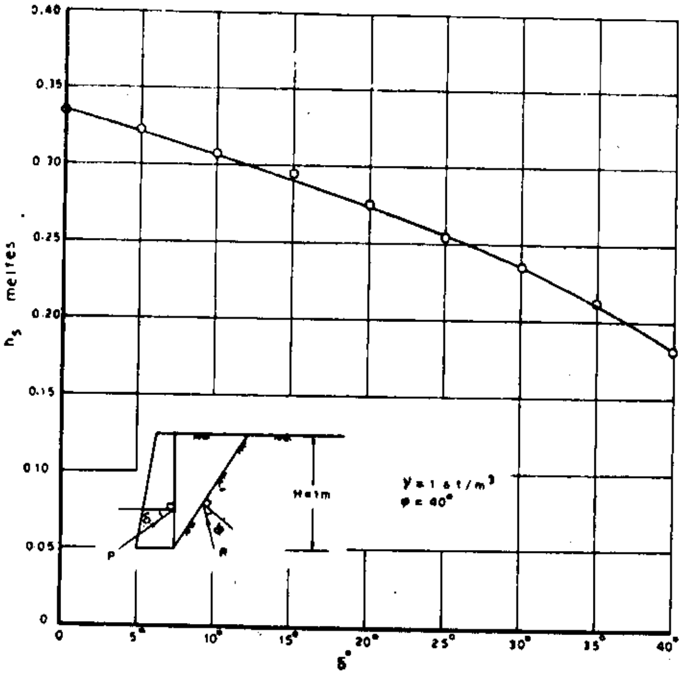

Fig. 4. Relationship between height of point of action of static earth pressure (h_s) with angle of wall friction ($\delta°$)

EFFECT OF VARIOUS PARAMETERS ON THE POINT OF APPLICATION OF DYNAMIC INCREMENT IN ACTIVE EARTH PRESSURE FORCE.

By using the proposed method of analysis, the effect of angle of wall friction (δ); angle of surcharge (i); the angle of wall back with vertical (α); the coefficient of horizontal acceleration due to earthquake (α_h) and the angle of shearing resistance of the soil over the

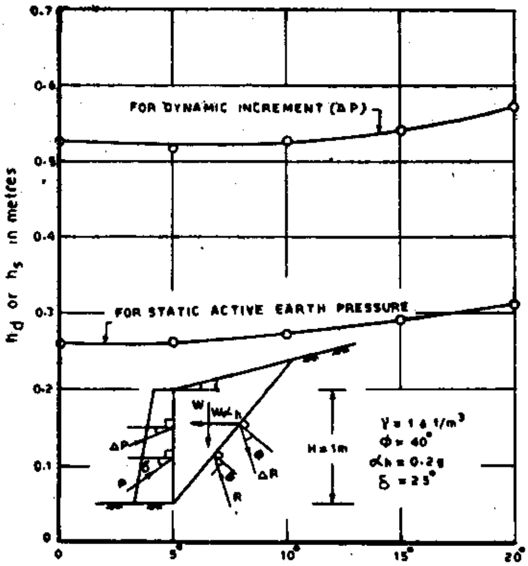

Fig. 5. Relationship between height of point of application of static earth pressure (h_s) and its dynamic increment (h_d) with angle of surface (2)

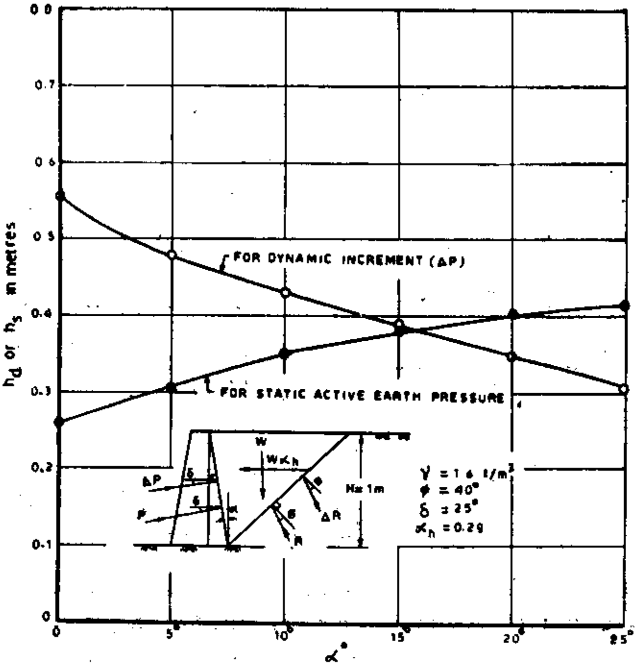

Fig. 6. Relationship between height of point of action of static active earth pressure (P) and its dynamic increment with angle of wall back (α°)

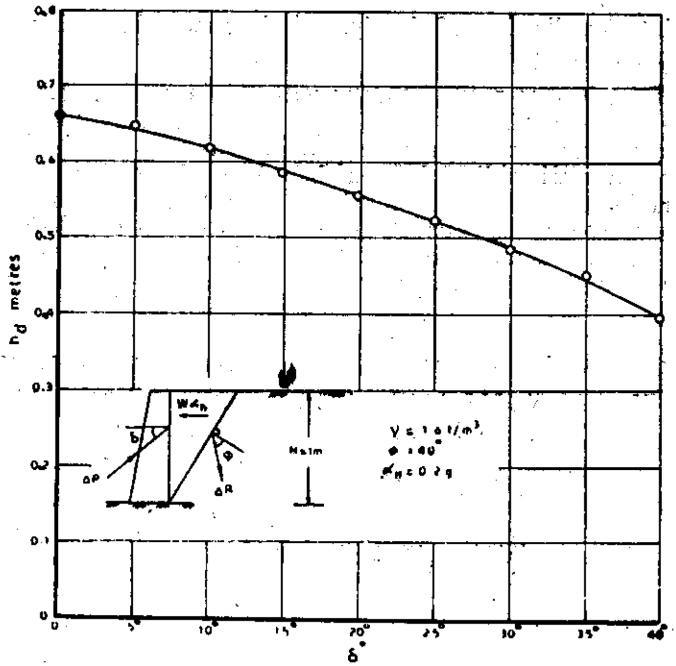

Fig. 7. Relationship between height of point of application of dynamic increment (h_a) with the angle of wall friction ($\delta°$)

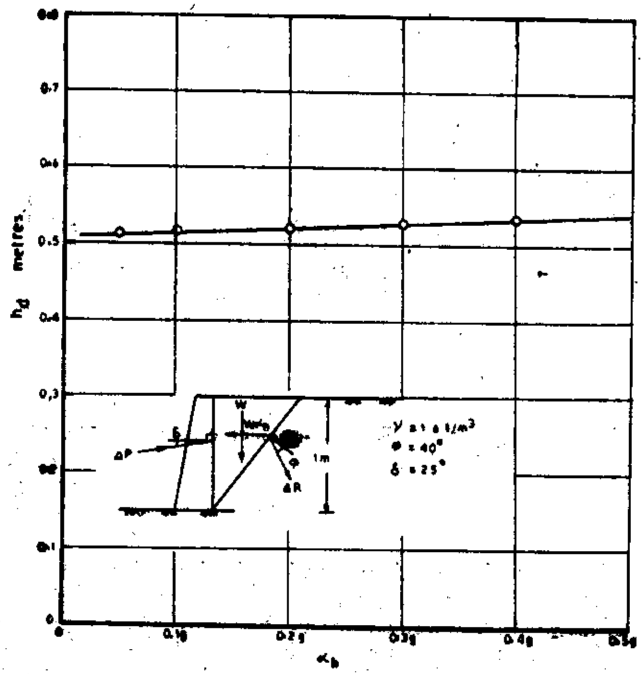

Fig. 8. Relation between height of point of application of dynamic increment (h_a) with horizontal acceleration (α_h)

height of the point of application of the dynamic increment in earth pressure force ($\triangle P$) above the base (h_d) are studied.

The value of h_d increases parabolically with the increasing values of δ; but for a smooth wall ($\delta=0$) h_d is equal to two third height of the wall above base; Fig. 7. But for very rough walls, the value of h_d may fall to as low a value of 0.4 H above base; thus showing that δ has a marked inuflence on h_d. From Fig 6, it is seen that h_d again decreases with the increasing angle of wall back with vertical (α) for wall leaning away from the fill. On the other hand increasing values of surcharge angle (i) tend to increase the value of h_d (Fig. 5). As far as horizontal acceleration coefficient α_h is considered, h_d increases linearly with increasing values of α_h, though the variation is not very sharp (Fig. 8).

COMPARISON OF THE FINDINGS WITH THE OTHER THEORIES, EXPERIMENTAL INVESTIGATIONS AND THE I.S. CODE OF PRACTICE.

The first theory to predict the dynamic active earth pressure was put forward by Mononobe (6) and Okabe (8) which is essentially the same as this proposed method, except that they assumed the dynamic earth pressure force to act at a hight of one third height above the base. They also assumed a different failure wedge under dynamic conditions. Based upon the moment equilibrium, Prakash and Basavanna (10) suggested an improvement for this method in the form, of a correction factor "C_{ha}", which predicted the total active dynamic earth pressure force to act at a height more than one third height above base (Fig. 9). Jacobsen (3) made a series of tests on 3′ (91.44 cm) high wall backfilled with dry sand and subjected to harmonic base excitations. His test data agreed well with the magnitude of dynamic active earth pressure force predicated by Mononobe and Okabe method only for values of $\alpha_h \leqslant 0.4$ (Fig. 10). But the dynamic increment ($\triangle P$) was found to act at a height of $\frac{2}{3}$H above base. A comprehensive investigation of a similar type was conducted by Ishii, Arai, and Tsuchida (1) using a shaking table of about 13 feet (3·96 m) in length 7 feet (2·13 m) in width and 2 feet (0·61 m) in depth. Their findings indicated that the rupture mass under dynamic condition practically behaved like a rigid mass; which supports the rigid body mechanics assumed by the proposed method and the Mononobe-Okabe method. Though the total dynamic active earth pressure force was found to be somewhat

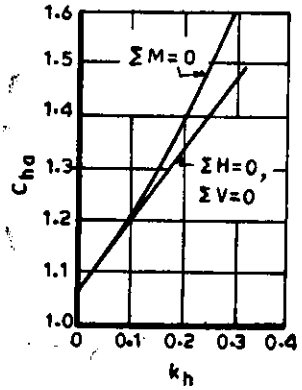

Fig. 9. Values of C_{ha} based on moment and force equilibrium conditions (After Prakash and Basavanna)

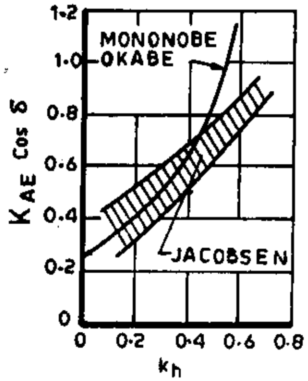

Fig. 10. Results of model tests by Jakobsen

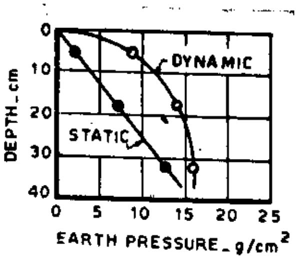

Fig. 11. Effect of base motion on earth pressure distribution (After Matsuo)

less than that predicted by Mononobe and Okabe method, the point of application was about 0·33 H to 0·4 H, above the base. The test results obtained by Mutsuo (5) show that the distribution of dynamic earth pressure behind retaining wall is curvilinear, as shown in Fig. 11.

From the study of the avialable experimental evidences, it can be concluded that though the dynamic active earth pressure force is found to act at a height greater than H/3 from the base, there is no unique value arrived at and the value normally varies between 0·33 H to 0·66 H above the base. This is understandable because the various parameters influencing the point of application of the same were different in different test setups. The point of application predicted by the proposed method is more rational in the sense, it takes into consideration the effect of various parameters quantitatively in predicting the point of application of this force, which again falls between 0·33 H to 0·66 H, abve the base; and as such agrees reasonably with the experimental data available.

The I. S. Code (2) suggests that the dynamic increment should be taken to act at a height of 0·66 H above the base. This is the extreme case and in most of the cases this clause is likely to overestimate the overturning moment due to dynamic increment ($\triangle P$)

THE VARIATION OF COEFFICIENT OF DYNAMIC INCREMENT WITH THE HORIZONTAL ACCELERATION COEFFICIENT.

The coefficient of the dynamic increment ($\triangle K_{AE}$) increases very markedly with increase in the value of coefficient of horizontal acceleration, α_h, particularly at larger values of α_h as suggested by Mononobe and Okabe. Seed (12) has arbitrarily suggested that for all practical purposes $\triangle K_{AE}$ is given by the linear expression

$$\triangle K_{AE} = (3/4)\alpha_h$$

The values of $\triangle K_{AE}$ as obtained by the proposed method of analysis, along with those

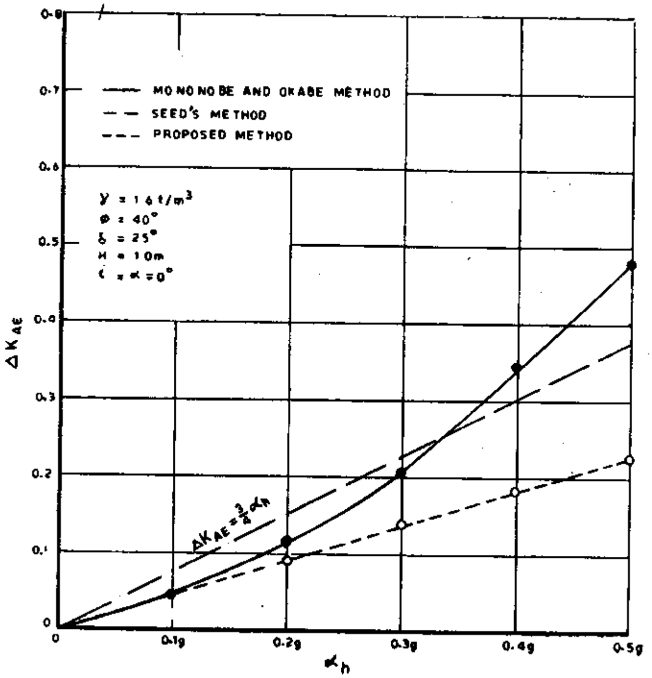

Fig. 12. **Relationship between dynamic increment in earth pressure coefficient $\triangle K_{AE}$ with horizontal acceleration (α_h)**

suggested by Mononobe and Okabe, and Seed are plotted in Fig. (12). The experimental results obtained by Jakobsen show that for higher values α_h ($\alpha_h > 0.4$) the predicted magnitude of earth pressure calculated by Mononobe and Okabe method is much in excess of the actual values observed. Though Seed's values are smaller than those suggested by Mononobe and Okabe for $\alpha_h > 0{\cdot}33$, it happens to be arbitrary in nature tending to overestimate the values of K_{AE}. In fact, the proposed method happens to be the only method which considers the influence of various parameters in determining the magnitude of the dynamic earthpressure.

However, a more rigorous check up with the experimental results taking into consideration the various parameters listed in this proposed method is required for further confirmation of the validity of the predicted results. Moreover, only a limited number of graphical solutions are made use of in studying the effect of various parameters on the point of application of the static and dynamic earth pressure. It would be very useful for the design engineers if an earth pressure table is prepared considering the variables in the range of practical interest.

CONCLUSIONS

A method has been proposed for determining the point of application of the dynamic increment of the earth pressure, assuming that the size of the failure wedge under dynamic condition will be the same as that under the static condition, if a failure wedge is already formed before the application of dynamic loads. The method uses all the three conditions of equilibrium in the analysis.

The results of the analysis show the following significant findings.

(i) The height of point of application of dynamic increment of earth pressure above base of wall (h_d) is always greater than that of static earth pressure force.

(ii) The value of h_d is a function of angle of wall friction (δ): angle of surcharge (i), angle of wall back with the vertical (α): coefficient of horizontal acceleration (α_h): and the value of angle of shearing resistance of the soil (ϕ).

(iii) The value of h_d decreases quite sharply with the increasing values of angle of wall friction (δ).

(iv) The value of h_d increases, but not very sharply with the increasing values of angle of surcharge (i).

(v) For the wall leaning away from the backfill, the value of h_d decreases; rather sharply, with increasing values of inclination of the wall (α).

(vi) The value of h_d increases linearly with the increasing values of horizontal acceleration (α_h).

(vii) The values of $\triangle K_{AE}$ predicted by Seed and by the proposed method are both linear functions of α_h. But Seed's method does not consider the effect of various variables that the proposed method considers. Also the values of $\triangle K_{AE}$ predicted by Seed's method are much larger than those calculated by the proposed method for those values of ϕ, δ, α, i, α_h and γ adopted in this analysis.

For the same set of analysis, the values of $\triangle K_{AE}$, predicted by Mononobe-Okabe method are smaller than those by Seed's method for $\alpha_h \leqslant 0.33$ g and are larger for $\alpha_h \geqslant 0.33$ g: the values of $\triangle K_{AE}$ from the present method are nearer to those predicted by Mononobe-Okabe method for $\alpha_h \leqslant 0.25$g, which covers the range of accelerations of maximum interest to the earthquake engineers.

ACKNOWLEDGEMENTS

The authors express their sincere thanks to Dr. A.S. Arya Professor and Head, School of Research and Training in Earthquake Engineering, University of Roorkee, Roorkee for the encouragement and valuable suggestions in writing this paper.

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NOTATIONS

a = length of the plane rupture surface behind the retaining wall.
H = height of the retaining wall measured from the base of the wall.
h_d = height of the point of application of the dynamic increment ($\triangle P$), measured vertically above the base of the wall.
h_s = height of the point of application of the static active earth pressure force (P) measured vertically above the base of the wall.
i = angle of surcharge (Fig. 1).
$\triangle K_{AE}$ = dynamic increment in earth pressure coefficient $= K_{ADynamic} - K_{Astatic})$
P = static active earth pressure force.
P_{dyn} = dynamic active earth pressure force.
$\triangle P$ = dynamic increment in active earth pressure force due to earthquake ($= P_{dyn} - P$).
R = Soil reaction.
$\triangle R$ = dynamic decrement in soil reaction due to earthquake.
W = weight of the soil mass in the failure wedge.
x = soil reaction at a distance measured along the rupture plane from the top end.
y = maximum soil reaction at the foot of the rupture plane.
Z = the distance measured from the top end of the rupture plane, at which the dynamic decrement in soil reaction is maximum.
α = angle of the wall back with respect to vertical for the wall leaning away from the fill.
α_h = coefficient of horizontal acceleration due to earthquake.
γ = unit weight of the soil.
δ = angle of wall friction.
ζ = inclination of rupture plane with respect to horizontal direction (Fig. 1).