Paper No. 143, Bull ISET, Vol. 11 No. 2, June 1974, pp. 33–37

ATTENUATION OF LOVE WAVES IN THE PRESENCE OF SINGLE SURFACE VISCOELASTIC LAYER

S. M. G. SUBHASH,[1] P. N. AGRAWAL,[2] V. K. GAUR[3]

INTRODUCTION

The amplitude attenuation with depth of the surface waves both Love and Rayleigh waves, in elastic earth model is of exponential nature (Stonley, 1958). Studies on the effect of viscoelasticity on the surface wave attenuation with depth have so far not been reported.

The applicability of the results for elastic earth models to the actual problems may be limited since the stress wave propagation in most solids is associated with the process of energy dessipation, [Kolsky, (1960), Knopoff (1969)]. This process, for many of the earth materials can be represented by linear viscoelastic models. Therefore, for present investigations, a simple model with single surface layer exhibiting Voigt type of visco-elasticity overlying an elastic half space was taken for the study of amplitude attenuation of Love waves of varying wave lengths, ranging between 0.3 to 2.4 times the thickness of surface layer. Amplitude attenuation has been studied upto depths, from free surface, which are whithin two times the thickness of surface layer.

Similar studies for models approximating, more to the real earth, may permit a better understanding of the attenuation of earthquake caused ground motions with depth.

FORMULATION OF THE PROBLEM

The geometry of the model chosen for the study, that is a viscoelastic layer of finite thickness overlying a semi-infinite elastic media, is shown in figure 1. The layered structure is referred to rectangular coordinate system with Z-axis perpendicular to the interface and directed into the semi infinite elastic media. Plane Z=0 represents the interface between the surface viscoelastic layer and the underlying half space; and the plane Z=−H represents the free surface. Let μ, μ' and ρ be the rigidity, conefficient of solid

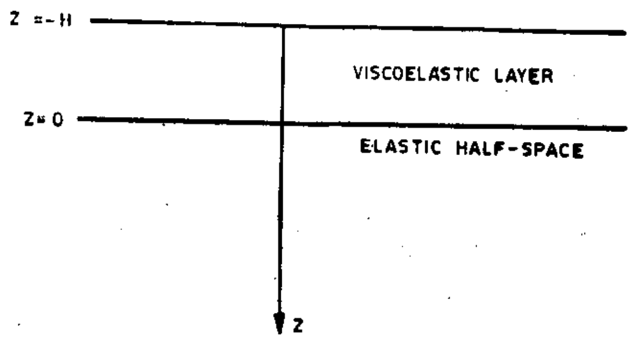

Fig. 1. Geometry of the Problem

[1]Lecturer in Seismology, School of Research and Training in Earthquake Engineering, Roorkee, (India).
[2]Reader in Seismology, School of Research and Training in Earthquake Engineering, Roorkee, (India).
[3]Professor in Geophysics, University of Roorkee, Roorkee (India).

viscosity and density values respectively, in the surface viscoelastic layer : μ_1 and ρ_1 be the rigidity and density values in the underlying elastic half space.

Neglecting the body forces and assuming small deformations, the equation of motion of Love waves propogating in the direction of X-axis in the two mediums can be written as (Ewing, Press, Jardetzky; 1957):

$$\left(\mu+\mu'\frac{\partial}{\partial t}\right)\left[\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Z^2}\right]=\rho\,\frac{\partial^2 V}{\partial t^2} \quad \text{...(1)}$$

$$\mu_1\left[\frac{\partial^2 V_1}{\partial X^2}+\frac{\partial^2 V_1}{\partial Z^2}\right]=\rho_1\,\frac{\partial^2 V_1}{\partial t^2} \quad \text{...(2)}$$

where V and V_1 are the *y*-components of displacement in the Voigt type of Viscoelastic medium and the elastic half space, respectively. For a plane harmonic wave, of wave length $2\lambda/k$ and wave velocity ω/k, these displacements V and V_1 can be represented by the following relations:

$$V=\dot{V}(Z)\exp\left[i\{(k+i\alpha)X-\omega t\}\right] \quad \text{...(3)}$$

$$V_1=V_1(Z)\exp\left[i(kX\times\omega t)\right] \quad \text{...(4)}$$

where $V,(z)$ and $V_1(z)$ are real functions of z, and α represents the damping coefficient in the surface viscoelastic layer. Substituting these values of V and V_1 in equations (1) and (2) respectively;

$$\frac{d^2V}{dZ^2}+\left[\frac{\rho\omega^2}{\mu-i\omega\mu'}-(k+i\alpha)\right]V=0 \quad (5)$$

$$\frac{d^2V_1}{dZ^2}+k^2\left[\frac{\omega^2}{k^2\beta_1^{\,2}}-1\right]V_1=0 \quad (6)$$

in equation (6) the quantity $\sqrt{\frac{\mu_1}{\rho_1}}$ has been replaced by β_1 i.e. the shear wave velocity in the elastic medium

The solutions for equations (5) and (6) as obtained in the study are given as follows :

$$V=\left[A\exp(irk\,Ze^{i\phi/2})+B\exp(-irk\,Ze^{i\phi/2})\right] \quad (7)$$

$$V_1=C\exp\left[ikZ\sqrt{\frac{\omega^2}{k^2\beta_1^{\,2}}-1}\right] \quad (8)$$

where ;

$$\phi=\tan^{-1}[\{pq-2a(1+q^2)\}/\{p-b(1+q^2)\}]$$

$$\gamma=\sqrt{\{pq-2a(1+q^2)\}^2+\{p-b(1+q^2)\}^2}$$

$$a=\alpha/k$$

$$b=1-a^2$$

$$q=\omega\mu'/\mu$$

and $p=\rho\omega^2/k^2\mu$

Using the boundary conditions i.e. (i) stress is zero at the free surface $Z=-H$, and (ii) the displacements and stresses at the interface $Z=O$ are continuous; the constants A and C are eliminated from the equations (7) and (8) to obtain following equation :

$$V=B\exp(irk\,Ze^{i\phi/2})[1+\exp\{2ik(H+Z)re^{i\phi/2}\}] \quad (9)$$

$$V_1=B\exp\left(irk\,Z\sqrt{\frac{\omega^2}{k^2\beta_1^{\,2}}-1}\right)[1+\exp(2irkHe^{i\phi/2})] \quad (10)$$

Thus the ratio of the amplitude of Love waves at any depth, to its amplitude at the free surface, in the surface viscoelastic layer (R_{VE}) and in the elastic half space (R_E) are as follows :

$$R_{VE} = \frac{V}{V_O} = \frac{\exp(-irk\, Ze^{i\phi/2})[1 + \exp\{2ik(H + Z)re^{i\phi/2}\}]}{2\exp(irkHe^{i\phi/2})} \tag{11}$$

$$R_E = \frac{V_1}{V_O} = \frac{\exp\left(ik\, Z\sqrt{\frac{\omega^2}{k^2B_1^2} - 1}\right)\left[1 + \exp(2ikrHe^{i\phi/2})\right]}{2\exp(irk\, He^{i\phi/2})} \tag{12}$$

Making substitution as follows :

$$R = \sqrt{\tfrac{1}{2}[\cos h2P + \cos 2Q]}$$

$$R_1 = \sqrt{\frac{1}{2}\left[\exp\left(-2k\sqrt{1 - \frac{\omega^2}{k^2\beta_1^2}}\right)\left\{\cos h2P' + \cos\ 2Q'\right\}\right]}$$

$$Q = \tan^{-1}[\coth 2P\ .\ \cos 2Q]$$

$$Q_1 = \tan^{-1}[\tan h\ P'\ .\ \tan Q']$$

and

$$P = rk(H + Z)\sin\phi/2$$

$$Q = rk(H + Z)\cos\phi/2$$

$$P' = kH\sin\phi/2$$

$$Q' = kH\cos\phi/2$$

expressions for R_{VE} and R_E can be rewritten as :

$$R_{VE} = Re^{i\theta}$$

$$R_E = R_1e^{i\theta_1}$$

Using equations (11) and (12), the amplitudes of Love waves upto a depth, twice the thickness of the surface layer and for wave lengths in the range of 0.30 to 2.4 times the surface layer thickness have been calculated. Elastic parameters used for the computation are given below, (Kanai, 1950).

$$\beta = 200 \text{ mts/sec}$$

$$\rho = 1.5$$

$$\beta_1 = 633 \text{ mts/sec}$$

$$\mu' = 6\times 10^8 \text{ C.G.S.}$$

DISCUSSION OF RESULTS

The results i.e. the amplitude of Love waves at different depths below the free surface both in the viscoelastic layer and the underlying elastic medium have been graphically presented in Fig. 2. In general, the Love waves in the surface viscoelastic layer shows a slight decrease and then a marked increase in their amplitudes. With H/λ ratio increasing, the increase becomes more and more prominent. The point where the increase in amplitude starts, keeps on shifting towards the free surface. For H/λ ratio as 1.2 the point where amplitude starts increasing is approximately at a depth of 0.2 times the thickness of the viscoelastic layer. However, for value of H/λ equal to 0.3, the increase in amplitude is not observed. The point of inflection lies between H/λ ratios 0.3 and 0.6. A comparison of the amplitudes at a depth twice the thickness of the surface layer in elastic and visco-elastic models is given in Table I.

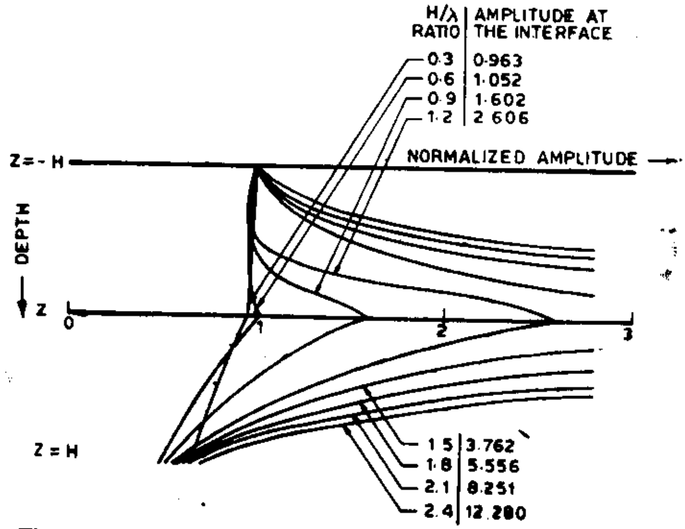

Fig. 2. Amplitude of love wave at different depth from the free surface, for different H/λ ratios

TABLE—I

Comparison of the amplitudes at a depth twice the thickness of the surface layer, in elastic and viscoelastic models.

H/λ	Amplitude of Love wave at a depth twice the thickness of surface layer	
	Elastic	Viscoelastic
0.3	.6378	.0669
0.6	.3341	.5048
0.9	.1203	.5319
1.2	.0027	.5992
1.5	.0817	·5990
1.8	.0719	.6126
2.1	.0578	.6300
2.4	.0520	.6495

CONCLUSIONS

On the basis of the results obtained in this study it can be concluded that there exists a critical value of H/λ ratio such that for values smaller than that critical value the viscoelastic layer can fairly be represented by an equivalent elastic layer for the purpose of attenuation of Love wave amplitudes. For H/λ ratios exceeding 0.6, the departures from the equivalent elastic model are significant.

ACKNOWLEDGEMENTS

The authors are thankful to Dr. A.S. Arya, Professor and Head, School of Research and Training in Earthquake Engineering, University of Roorkee, for the permission to publish this paper.

The first author received financial support from a grant from National Oceanic and Atmospheric Administration's Special Foreign Currency Research and Development Programme.

REFERENCES

1. Ewing, Press and Zardetzky, 1957. 'Elastic Wave in Layered Media', McGraw Hill Book Co., New York.
2. Gutenberg, B., 1958, Attenuation of Seismic Waves in Earth's Mentle, Bull. Seism. Soc. Am. Vol. 48.
3. Kolsky, H., 1960. Viscoelastic waves, International Symposium on Stress Wave Propagation in Materials.
4. Kanai, K., 1950. The Effect of Solid Viscosity of Surface Layer, on Earthquake Movement, Bull. Earthquake Research Inst., Tokyo.
5. Knopoff, L., 1969, Attenuation of Seismic waves in the mantle, "The Earth's crust and Upper Mantle", Geophysical Monogarph '13' of American Geophysical Union.
6. Stonley, R., 1958. The Variation of Amplitude and Energy with Depth in Love Waves, Contribution in Geophysics, Vol. 1 Pergamon Press London.