Paper No. 124, Bull. I. S. E. T., Vol. 9, No. 4, December 1972, pp. 121–128

VIBRATION STUDIES OF A SURGE SHAFT SLAB

A.S. ARYA* AND S.K. THAKKAR*

INTRODUCTION

Vibrations of slabs in the surge shaft chambers of hydroelectric power stations are often caused due to dynamic effect of upsurge and down surge conditions and water hammer effects due to sudden closing or opening of the gates. It is generally feared that if the frequencies of the dynamic forces match with one of the natural frequency of the slab, a resonance condition may arise which may lead to severe amplification of forces. The problem of computation of natural frequency is complicated by the fact that the slab being submerged in water, a virtual mass of water will act with it which is expected to modify its frequency considerably. This contribution of the water can only be determined by model test. This paper presents the results of an investigation carried out by the authors in regard to the vibrations of Surge-shaft slab of Dehar power plant in the Beas-Sutlej Link Project.

THE STRUCTURE

The general arrangement of water carrier tunnel, the surge-shaft chamber and its top slab is shown in Fig. 1. The clear diameter of the slab is 22.9m and thickness 2.14m. There are three port openings of 3.66m × 1.07m each in the slab. The slab is anchored to the rock through rock bolts and it is to be designed for the following two loading conditions:

(1) A differential water head of 73.2m acting upward from below in case of worst "upsurge" condition.

(2) A differential head of 33.6m acting downward from above in case of worst "down-surge" condition.

(3) A load of 4.57m of rock is to be taken as if suspended from the slab through rock bolts when the upsurge condition occurs. The same load is assumed to act in estimating the lowest natural frequency of slab.

Figures 2 and 3 show the results of upsurge and down surge studies. The slab is also subjected to water hammer corresponding to gate closing time of 6.5 seconds. Figure 4 gives the maximum pressure rise wave at the slab location due to water hammer (1).

METHOD OF ANALYSIS

As seen from Fig. 1, the surge shaft slab is a circular slab of 22.9m over all diameter having a cut of 7.62m diameter at one end, a tapering thickness from 5.18 to 2.14m in the portion where the inlet tunnel joins the chamber, and three rectangular openings of 3.66m × 1.07m where the penstock tunnels take off. The slab is monolithic with the shaft lining as well as the riser wall and is assumed to have fixed boundary condition there. The main problem was to determine the natural frequency of the slab of varying thickness and having an irregular fixed boundary. The natural frequency of such a complex system cannot be determined in a closed form and numerical technique had to be used for obtaining the results. The finite difference approach was used for obtaining the frequency determinant and computing the natural frequencies and mode shapes of the slab. To have a thorough check of the numerical work, the same finite difference operators as were used for the actual slab conditions, were also used for finding the deflections and moments under uniform load and also frequencies of solid uniform circular slab for which

*Professor and Reader respectively, School of Research and Training in Earthquake Engineering, University of Roorkee, Roorkee, U.P.

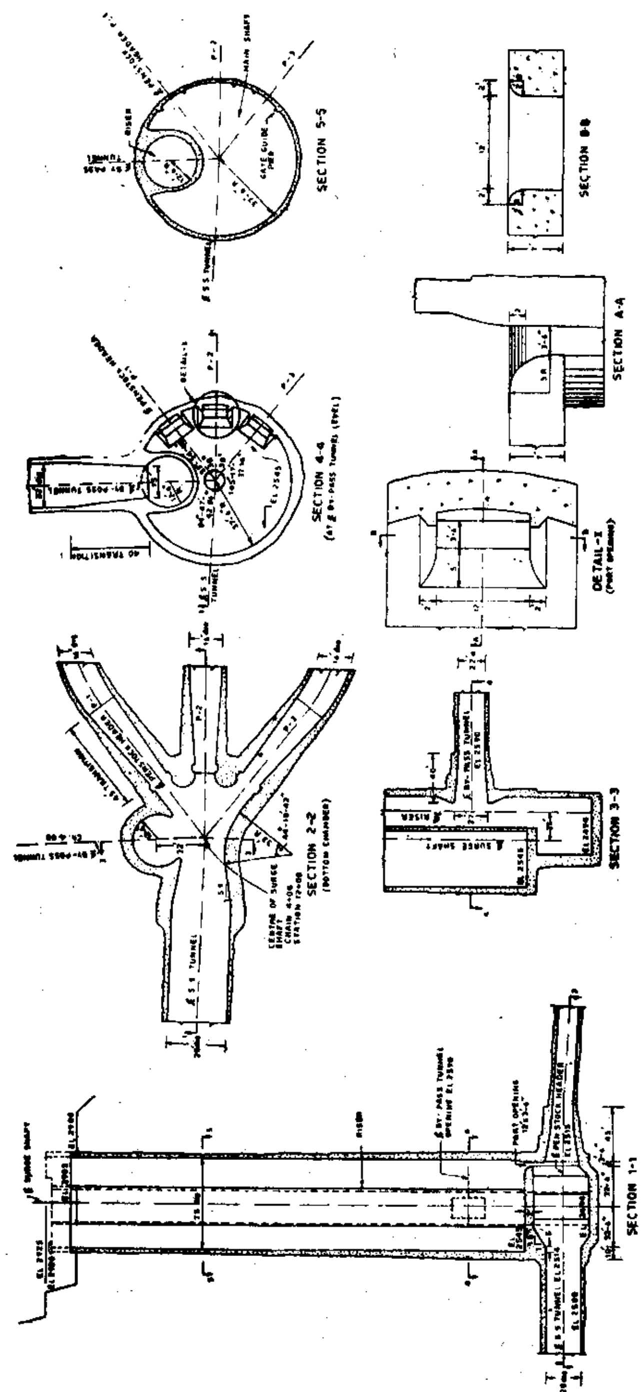


Fig. 1. General Arrangement of Dehar Surge Shaft.

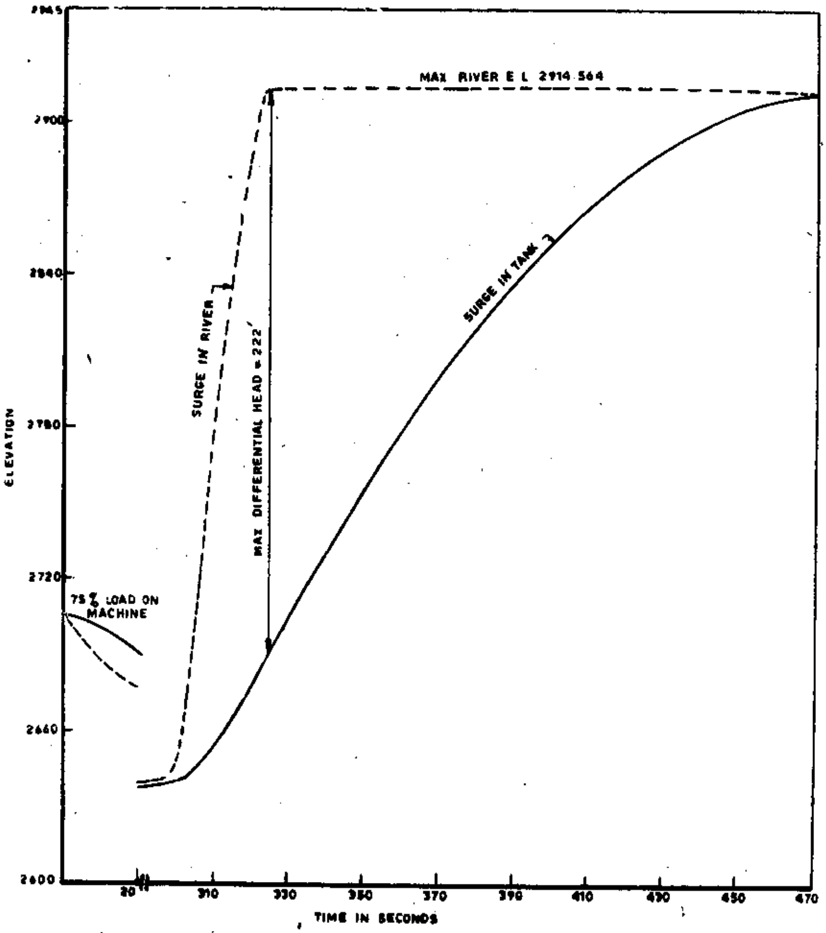


Fig. 2. Surge for Loading Condition 75% to 100% to 0%

accurate results are available. A close agreement in the results was found indicating that the method and expressions could be used for the complex slab with confidence.

FINITE DIFFERENCE PATTERN

The slab was divided into a rectangular grid as shown in Fig. 5. The general finite difference pattern for an internal point of the slab is shown in Fig. 6 whereas the expressions for various terms a_{11} to a_{88} are given in Appendix I. This pattern is a general one which could take care any elemental distances between the grid lines as indicated by the fractions r_1 to r_8 (2).

BOUNDARY CONDITION

The slab being fixed at the boundary two conditions can be applied at each boundary point, namely, (i) deflection at the boundary is zero and (ii) the slope of the deflected slab normal to the boundary can be replaced by a series of straight lines forming a large size polygon, the slope of the surface along the tangent to the boundary may also be assumed to be zero. Thus the slope of the surface may be taken equal to zero at the boundary in any direction. This condition was used in writing down the finite difference

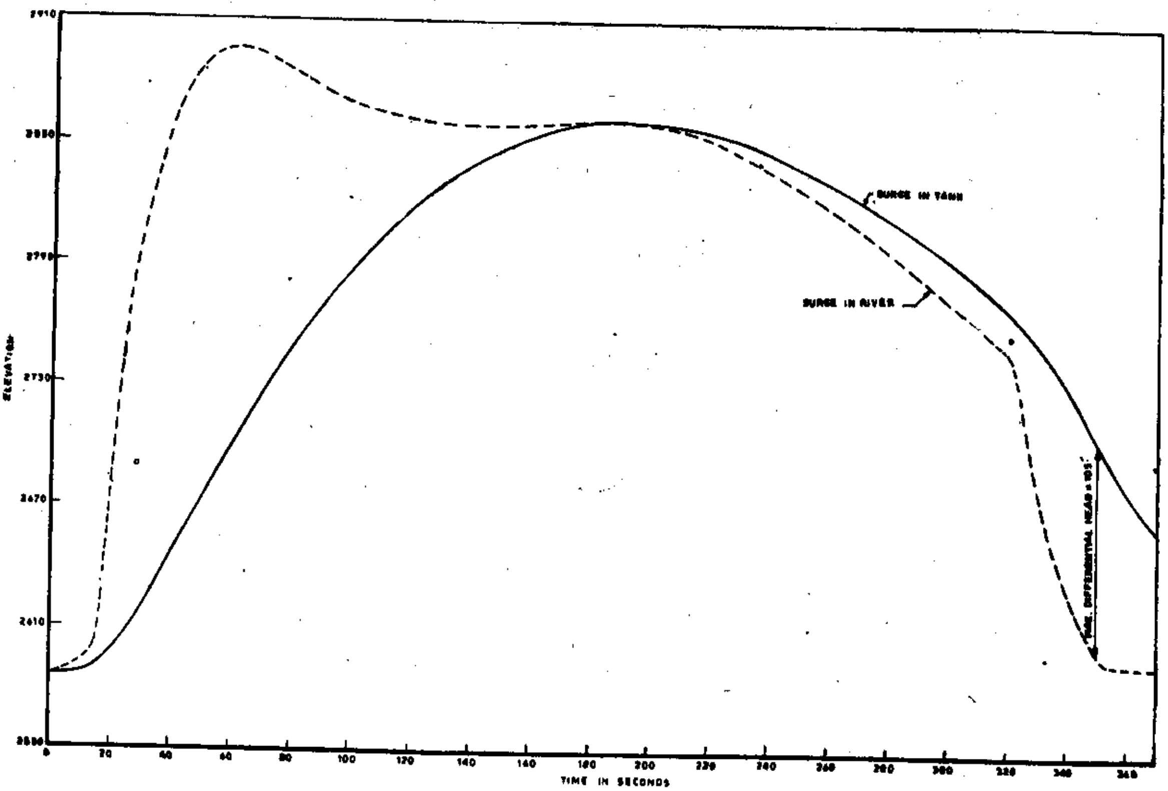


Fig. 3. Surge for Loading condition 100% to 0% to 50%.

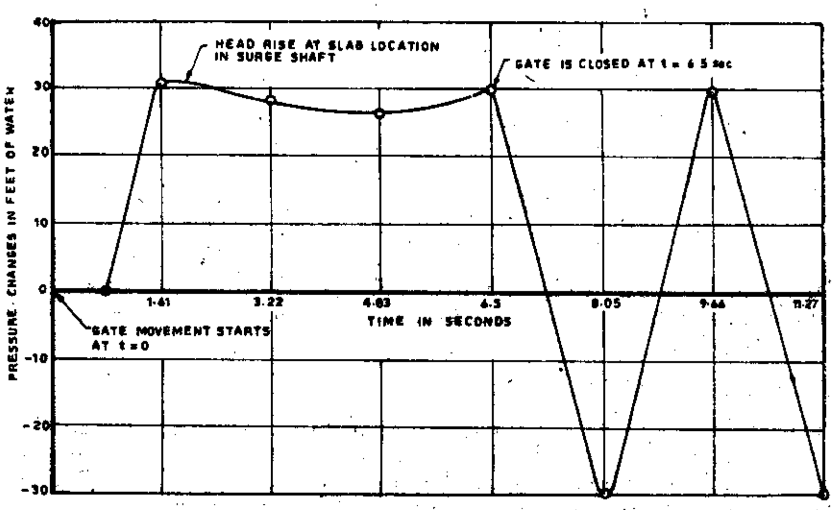


Fig. 4. Time History of the Pressure Changes due to Water Hammer at the Slab Location in Surge Shaft.

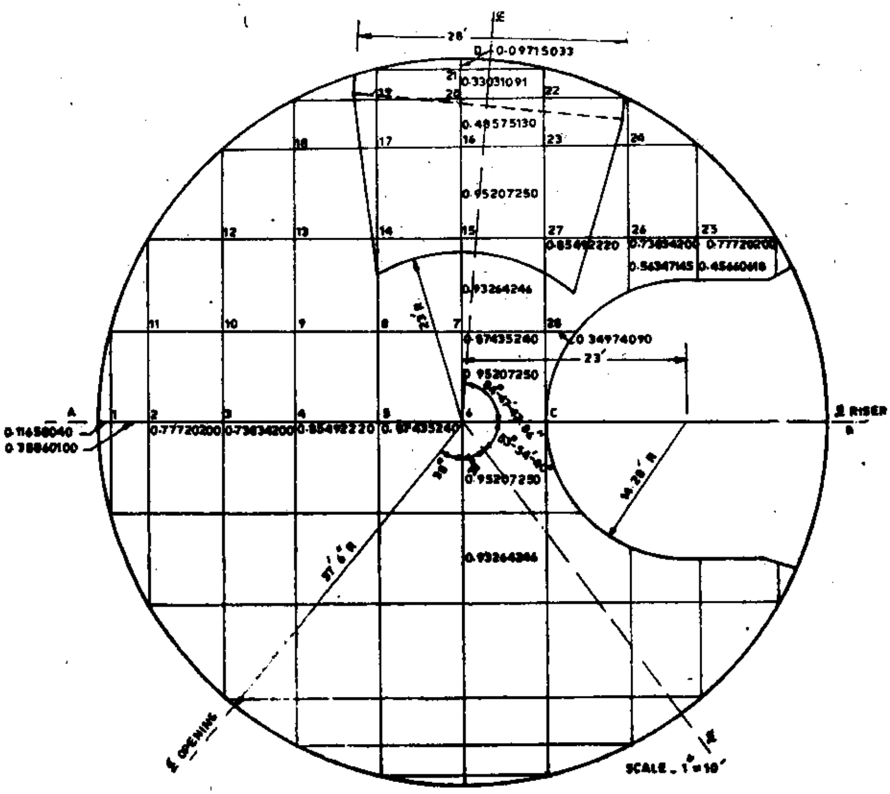
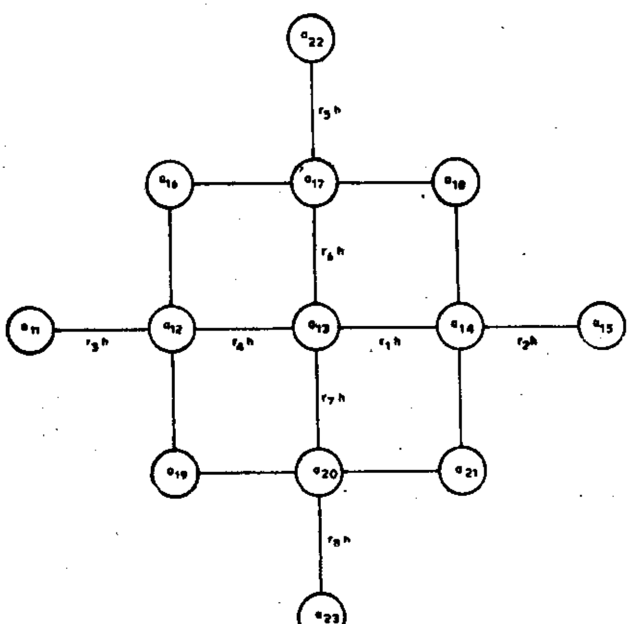


Fig. 5. Rectangular Grid Pattern for Circular Slab



$$= \frac{q}{D}$$

where q = load per unit area on the slab
D = flexural rigidity of slab per unit width

Fig. 6. Finite Difference Pattern for an Internal Point

patterns at points near boundary and for eliminating the imaginary points beyond the boundary.

EQUILIBRIUM EQUATIONS AND NATURAL FREQUENCY

Applying the finite difference operators to the various grid points within the slab a sufficient number of equations was written in terms of the deflections at the grid points. In determining the natural frequency and mode shape in any mode, the applied load was replaced by the inertia forces of the mass of the slab which is a function of the deflection at the point under consideration. Thus the equations of equilibrium at all grid points result in an equal number of homogenous simultaneous equations. From the frequency determinant of the coefficients in these equations, the natural frequencies and mode shapes have been derived.

ANALYTICAL RESULTS

Three cases of slab were considered as follows:

(a) Uniform circular slab fixed at edges (symmetrical about XX)

(b) Uniform circular slab with circular cut to riser (symmetrical about XX)

(c) Non uniform circular slab with circular cut due to riser (symmetrical about XX). The effect of rectangular holes was neglected in view of their small size as compared with the size of the slab.

The fundamental frequency and mode shapes for the above three cases are given in Fig. 7 and the frequency values are 18.48, 31.55 and 38.09 c/s respectively. The corres-

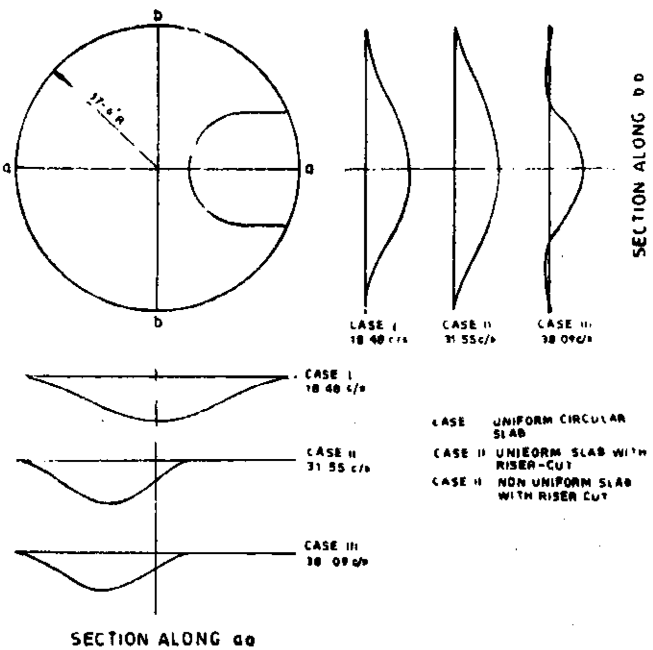


Fig. 7. Mode Shape and Frequencies of Surge Shaft Slab.

ponding frequencies for the case of rock load suspended from the slab will be 10.5, 17.93 and 21.7 cycles per second.

MODEL TESTS

The model was made in perspex to a scale of 1/39 of the prototype radius. First the slab was kept without hole or risers. Natural thickness of sheet was used which represented the thickness of slab to 1/710 scale. The slab was rigidly connected to the container walls by gluing it with chloroform. The frequency of the slab was determined in air in free vibrations and found to be 25 c/s. When extrapolated to the prototype through dimensional relationships, the prototype frequency of solid uniform 2.14m thick slab would be 18.1 c/s which compares very closely with the value of 18.48 c/s found by the finite difference solution. For this case closed form solution is also available and the frequency by that works out as 18.81 c/s. Thus the experimental and analytical results show very close corroboration.

Next this model was modified to incorporate the riser and port openings so as to simulate the required condition. The slab was still uniform in thickness and the increase in thickness was not incorporated. Since the increase in thickness increases the frequency, the uniform condition was on conservative side. The model slab was tested for vibration in air as well as under water. The model frequencies of uniform slab were found as 27.8 c/s in air and 2.04 c/s in water. The large reduction of frequency under water occurs due to a mass of water which virtually vibrates with the slab. The equivalent depth of the virtual mass of water was found to be 1.14 times the diameter of the slab. When the same ratio is assumed for calculating the frequency of the prototype slab, and the experimental frequencies are extrapolated to the prototype, the fundamental frequency under various conditions are found to be as follows:

In air

For self weight of uniform slab, f = 20.1 c/s

For slab+rock load = 11.4 c/s

Under water

For slab+rock load+virtual mass of water, f = 6.92 c/s

Thus the lowest value of fundamental frequency is indicated to be 6.92 c/s. Actually whole rock mass will perhaps not be dislodged from the rock and suspended from the slab. Thus the frequency of 6.92 c/s will be a conservative value.

AMPLIFICATION OF PRESSURES

From the results presented above it is seen that the lowest possible frequency of the surge shaft slab could be 6.92 c/s. From Fig. 4 the frequency of the water hammer is found to be 0.31 c/s. It is thus seen that the frequency of water hammer pressure pulse is only 0.0447 of the frequency of the surge shaft slab. In a case like this, no appreciable dynamic amplification is obtained. Therefore, the water pressure under various conditions may be assumed to be applied to the slab statically for design purposes.

CONCLUSION

(1) The analytical frequency of uniform circular slab computed by the finite difference pattern differs from the analytical closed form solution by 1.8% and from the experimental value by 2% thus proving the suitability of the method adopted.

(2) The virtual mass effect of water on the frequency of the slab is substantial. In the case studied herein, it is equal to the mass of water having a depth 1.14 times the diameter of slab. Thus it must be given due consideration.

(3) In the case of Dehar Surge shaft slab, the lowest probable natural frequency of the slab is more than 20 times the frequency in water hamer wave, hence no possibility of resonance. The dynamic amplification of the pressures is negligible.

ACKNOWLEDGEMENT

The study was proposed and financed by the Beas Sutlej Link-Project. Numerical computations were carried out on I B M 7044 computer at I I T, Kanpur.

REFERENCES

1. Wang, Ping-Chun, "Numerical and Matrix Methods in Structural Mechanics", John Wiley & Sons Chap. 1, 1965.
2. Arya, A. S. and Thakkar, S. K., "Vibration Studies of Top Slab of Dehar Surge Shaft Slab", Report of School of Research and Training in Earthquake Engineering, 1971.

APPENDIX–I

Values of a_{11} to a_{23} for finite difference pattern for an internal joint.

$$a_{11}=\frac{24}{r_3\,(r_3+r_4)\,(r_1+r_3+r_4)\,(r_1+r_2+r_3+r_4)}\,w_{11}$$

$$a_{12}=\left[\frac{-24}{r_3r_4\,(r_1+r_4)\,(r_1+r_2+r_4)}-\frac{1}{r_4\,r_6\,r_7\,(r_1+r_4)}\right]w_{12}$$

$$a_{13}=\left[\frac{24}{r_1r_4\,(r_1+r_2)\,(r_3+r_4)}+\frac{24}{r_6r_7\,(r_7+r_8)\,(r_5+r_6)}+\frac{8}{r_1r_4r_6r_7}\right]w_{13}$$

$$a_{14}=\left[\frac{-24}{r_1r_2\,(r_1+r_3+r_4)\,(r_2+r_4)}-\frac{8}{r_1r_6r_7\,(r_1+r_4)}\right]w_{14}$$

$$a_{15}=\left[\frac{24}{r_2\,(r_1+r_2+r_3+r_4)\,(r_1+r_2+r_4)\,(r_1+r_2)}\right]w_{15}$$

$$a_{16}=\left[\frac{8}{r_4r_6\,(r_1+r_4)\,(r_6+r_7)}\right]w_{16}$$

$$a_{17}=\left[\frac{-24}{r_5r_6\,(r_6+r_7)\,(r_6+r_7+r_8)}-\frac{8}{r_1r_4r_6\,(r_6+r_7)}\right]w_{17}$$

$$a_{18}=\left[\frac{8}{r_1r_6\,(r_1+r_4)\,(r_6+r_7)}\right]w_{18}$$

$$a_{19}=\left[\frac{8}{r_4r_7\,(r_6+r_7)\,(r_1+r_4)}\right]w_{19}$$

$$a_{20}=\left[\frac{-24}{r_7r_8\,(r_5+r_6+r_7)\,(r_6+r_7)}-\frac{8}{r_1r_4r_7\,(r_6+r_7)}\right]w_{20}$$

$$a_{21}=\left[\frac{\quad}{r_1r_7\,(r_1+r_4)\,(r_6+r_7)}\right]w_{21}$$

$$a_{22}=\left[\frac{24}{r_5\,(r_5+r_6)\,(r_5+r_6+r_7)\,(r_5+r_6+r_7+r_8)}\right]w_{22}$$

$$a_{23}=\left[\frac{24}{r_8\,(r_5+r_6+r_7+r_8)\,(r_6+r_7+r_8)\,(r_7+r_8)}\right]w_{23}$$