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THE CONCEPT OF CUMULATIVE DUCTILITY STRENGTH SPECTRA AND ITS USE WITHIN PERFORMANCE-BASED SEISMIC DESIGN

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ABSTRACT

A seismic design procedure that does not take into account the maximum and cumulative plastic deformation demands that a structure will likely undergo during severe ground motion could lead to unreliable performance. Damage models that quantify the severity of repeated plastic cycling through plastic energy are simple tools that can be used for practical seismic design. The concept of constant cumulative ductility strength spectra, developed from one such model, is a useful tool for performance-based seismic design. Particularly, constant cumulative ductility strength spectra can be used to identify cases in which low cycle fatigue may become a design issue, and provides quantitative means to estimate the design lateral strength that should be provided to a structure to adequately control its cumulative plastic deformation demands during seismic response.

KEYWORDS: Low Cycle Fatigue, Damage Index, Plastic Energy, Strength Reduction Factor

INTRODUCTION

Current philosophy for seismic design of typical residential or commercial structures accepts the possibility that significant inelastic behavior will occur during severe seismic excitations. The mechanical characteristics of a structure deteriorate when deformations reach the range of inelastic behavior. Such deterioration can be important during long and severe ground motions, when several excursions into the inelastic range are expected. A possible consequence of deterioration of the hysteretic behavior of a structure is failure of critical elements at deformation levels that are significantly smaller than its ultimate deformation capacity. In this paper, this failure mode will be termed 'low cycle fatigue'.

Low cycle fatigue should be avoided, particularly for conditions that may result in repeated plastic cycling. The complexity of low cycle fatigue has resulted in significantly different opinions regarding how to account for it during seismic design. This paper discusses a set of simple tools recently developed for practical seismic design against low cycle fatigue. Although emphasis is placed on the design of reinforced concrete structures, the tools can be calibrated for other structural materials.

LOW CYCLE FATIGUE

Experimental and field evidence indicate that the strength, stiffness and ultimate deformation capacity of reinforced concrete elements and structures deteriorate during excursions into the plastic range of behavior. Excessive hysteretic degradation may lead to an excessive accumulation of plastic deformation that may lead to failure at deformation levels that are significantly smaller than the ultimate deformation capacity of the structure under uni-directional loading.

This phenomenon, denoted herein as low cycle fatigue, has been repeatedly observed in laboratory tests. For example, Panagiotakis and Fardis (2001) recently observed that the deformation at failure of reinforced concrete elements subjected to typical load-histories applied in laboratory tests can be estimated as 60% of their ultimate deformation capacity. Independently, Bertero (1997) recommended that the maximum ductility demand a structure undergoes during ground motion should be limited to 50% of its ultimate ductility.

The importance of plastic cycling on the deformation capacity of reinforced concrete structures has been known for some time. This effect caught the attention of several researchers during the 1970s when experimental studies were carried out on the cyclic response of reinforced concrete members and beam-column sub-assemblages. It was observed that the hysteretic behavior of ductile beams showed a tendency for degradation due to the presence, among other things, of flexural cracking, bond deterioration and shear effects (Bertero and Popov, 1977; Gosain et al., 1977; Scribner and Wight, 1980; Darwin and Nmai, 1985). As a consequence, these beams tended to eventually exhibit non-ductile behavior and even fragile failure. Several researchers discussed the need to account for the effect of cycling on the performance of earthquake-resistant structures. Some of the options that were visualized involved proportioning the beams to control the level of shear stress. Detailing schemes were formulated to enable structural elements to undergo several cycles of plastic deformation with stable hysteretic behavior (Bertero and Popov, 1977; Gosain et al., 1977).

In the 1980s and 1990s, the engineering profession confronted the need to design structures with predictable performance. Performance-based seismic design became a fundamental concept for the formulation of seismic design methodologies. As a consequence, proposals for design against low cycle fatigue began focusing on deformation control rather than relying exclusively on detailing recommendations to ensure stable hysteretic behavior. A key issue during the development of design methodologies to control low cycle fatigue was the recognition that the lateral strength of a structure plays an instrumental role in controlling the seismic demands that eventually induce this type of failure. Some researchers have suggested that there is no direct relation between strength and the level of seismic damage in structural elements, and that an increase in strength does not necessarily result in increased seismic safety (Priestley, 2000). Within the context of design against low cycle fatigue, it is important to emphasize that lateral strength is not supplied to enhance the deformation capacity of a structure, but as a mean of controlling maximum and cumulative plastic deformation demands, and avoiding uncontrolled and excessive degradation of its structural properties.

TARGET DUCTILITY

Target ductility is defined as the maximum ductility ($\mu_{\max}$) the structure can reach during the design ground motion before the level of structural damage exceeds a preset threshold. Within the context of low cycle fatigue, this threshold corresponds to incipient failure or collapse.

In general, it has been agreed that as the severity of plastic cycling increases, $\mu_{\max}$ should decrease with respect to the ultimate ductility (μ_u) the structure is able to undergo under monotonically increasing lateral deformation (unidirectional loading). How much smaller $\mu_{\max}$ should be with respect to μ_u (or how much bigger μ_u with respect to $\mu_{\max}$) depends on three variables: the value of the known ductility (either $\mu_{\max}$ or μ_u), a ground motion parameter that quantifies the severity of plastic cycling, and a structural parameter that characterizes the cycling capacity of the structure.

Using the concept of target ductility, two approaches can be considered for the formulation of a performance-based design methodology that accounts for low cycle fatigue (Teran-Gilmore and Jirsa, 2004). The first of these approaches (denoted herein 'Approach A') requires estimating $\mu_{\max}$ given that μ_u is known. That is, Approach A requires the estimation of a threshold value for the maximum plastic response in the structure given that its ultimate deformation capacity is known. The steps involved in Approach A can be schematized as follows:

- Define the type of detailing to be used in the structure (e.g., ductile versus non-ductile).
- Establish the value of the fundamental period of vibration (T) of the structure. The determination of T within the context of performance-based design has been discussed by Bertero and Bertero (1992), and Priestley (2000).
- Establish values to characterize the ultimate and cumulative deformation capacities of the structure. Note that these values depend on the type of detailing to be used.
- Establish $\mu_{\max}$ as a function of the severity of ground motion and of the ultimate and cumulative deformation capacities of the structure.
- Establish the design base shear that will allow the structure to control its maximum plastic demand within the threshold defined by $\mu_{\max}$.

ENERGY AS DESIGN REPRESENTATION OF CUMULATIVE LOADING

Significantly different methods have been proposed to estimate the severity of plastic cycling, and various design methodologies that account for the effect of low cycle fatigue have been offered. An option that has been considered attractive, due to its simplicity, has been the characterization of cumulative loading through energy concepts. Housner (1956) offered one of the earliest discussions regarding the need to consider explicitly the effect of plastic cycling through energy concepts. Later, several attempts have been made to estimate the energy demands in simple systems, and to offer insights on how to use these demands for design purposes (Zahrah and Hall, 1984; Kuwamura and Galambos, 1989; Akiyama and Takahashi, 1992).

Design for low cycle fatigue was advanced with the formulation and calibration of damage indices (Powell and Allahabadi, 1987; Cosenza et al., 1993), and the formalization of an energy balance equation for design purposes (Uang and Bertero, 1990). Based on these concepts, several design methodologies that account for low cycle fatigue have been formulated (Fajfar, 1992; Bertero and Bertero, 1992; Krawinkler and Nassar, 1992; Cosenza and Manfredi, 1996).

Today there are still significantly different approaches towards the formulation of a design representation for the energy demands. Some researchers suggest that energy spectra could be formulated and used for design purposes (Akiyama and Takahashi, 1992; Chou and Uang, 2000). Other options include accounting for cumulative loading in the structure through indirect measures of the plastic energy (Fajfar, 1992; Bertero and Bertero, 1992), and deriving the plastic energy demands from other relevant seismic demands (Teran-Gilmore, 1996; Decanini and Mollaioli, 2001).

The total plastic energy dissipated by a structure during an earthquake ground motion is denoted herein as $E_{H\mu}$. The plastic energy demand can be interpreted physically by considering that it is equal to the total area under all the hysteresis loops the structure undergoes during a ground motion. In this sense, $E_{H\mu}$ provides a rough idea of the cumulative plastic deformations in the structure. Nevertheless, $E_{H\mu}$ by itself does not provide enough information to assess structural performance. Thus, it is convenient to take into account simultaneously $E_{H\mu}$, and the strength and stiffness of a system, as follows:

$$NE_{H\mu} = \frac{E_{H\mu}}{F_y \delta_y} \tag{1}$$

where $NE_{H\mu}$ is the normalized plastic energy, and F_y and δ_y (shown in Figure 1(a)) are the yield strength and yield displacement, respectively. For an elasto-perfectly-plastic system subjected to a single plastic excursion (Figure 1(b)):

$$E_{H\mu} = \delta_p F_y = (\delta_c - \delta_y) F_y = \left(\frac{\delta_c}{\delta_y} - 1 \right) \delta_y F_y = (\mu_c - 1)\delta_y F_y \tag{2}$$

where δ_c is the cyclic displacement, δ_p is the plastic displacement associated with the plastic excursion, and μ_c, equal to δ_c/δ_y, is the cyclic ductility. The normalized plastic energy for the plastic excursion can be expressed as:

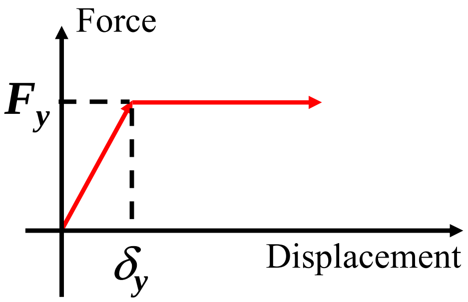
$$NE_{H\mu} = \frac{E_{H\mu}}{\delta_y F_y} = \frac{\delta_p F_y}{\delta_y F_y} = \frac{\delta_p}{\delta_y} = \mu_p \tag{3}$$

where μ_p is the plastic ductility reached in the excursion. Note that for a single plastic excursion, $NE_{H\mu}$ is a direct measure of the plastic displacement.

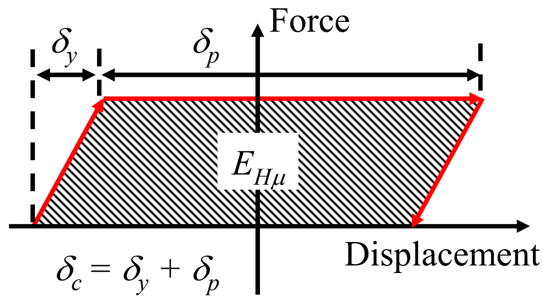
For an elasto-perfectly-plastic system subjected to multiple plastic excursions, $NE_{H\mu}$ is the sum of all plastic displacements reached in the different cycles normalized by δ_y, in such way that:

$$NE_{H\mu} = \frac{\sum_{i=1}^{Nexc} \delta_{pi}}{\delta_y} = \sum_{i=1}^{Nexc} \mu_{pi} \tag{4}$$

where δ_{pi} and μ_{pi} are the plastic displacement and plastic ductility, respectively, associated with the ith excursion, and *Nexc* is the total number of plastic excursions during the ground motion. Note that $NE_{H\mu}$ is a direct measure of the cumulative plastic displacement demands. For a system with degrading hysteretic behavior, $NE_{H\mu}$ could be defined to include all plastic excursions for which the capacity does not degrade to a value less than a specified fraction of F_y (say 0.75). Such a definition allows for rational evaluation of structural damage in reinforced concrete structures through the use of $NE_{H\mu}$.



(a) Properties at yield



(b) Plastic excursion and cyclic demands

Fig. 1 Definitions of strength and deformation quantities

Several researchers have discussed the concept of $NE_{H\mu}$ and its potential use during seismic design. For instance, Gosain et al. (1977) defined 'Work Index' as a measure of the energy absorbed at the hinging region of a structural element normalized by the product of the strength and displacement at first yield. Based on the influence of parameters, such as the shear span and axial compression, on the value of this index, they offered some recommendations for the design and detailing of ductile reinforced concrete elements. Later, other researchers used modified versions of the Work Index for similar purposes (Scribner and Wight, 1980; Darwin and Nmai, 1985). Recently, Krawinkler and Nassar (1992) formulated a seismic design methodology that explicitly considers the effect of plastic cycling by considering $NE_{H\mu}$ as a direct measure of the severity of the cyclic demands.

Table 1: Characteristics of Motions Included in 'Mexico Soft'

Record	**Date**	**PGA[1] (cm/sec^2)**	**PGV[2] (cm/sec)**	**T_s (sec)**
Alameda EW	04/25/89	46	15	2.1
Alameda NS	04/25/89	37	10	2.1
Garibaldi EW	04/25/89	52	17	2.2
Tlahuac EW	09/19/85	118	35	2.1
Tlahuac NS	09/21/85	49	13	2.0
Tlahuac EW	09/21/85	51	15	1.9
SCT EW	09/19/85	167	61	2.0

1 - Original Peak Ground Acceleration; 2 - Original Peak Ground Velocity

GROUND MOTIONS

To assess the effect of low cycle fatigue in the seismic performance and design of earthquake-resistant structures, four sets of ground motions with different energy content were considered. Three sets correspond to the Los Angeles (LA) urban area, and the fourth, to the lake zone of Mexico City. The ground motions for LA, established as part of the FEMA/SAC Steel Project (Somerville et al., 1997), were grouped in sets of twenty motions as follows: design earthquake for firm soil with 10% exceedance in 50 years (LA 10in50), design earthquake for firm soil with 50% exceedance in 50 years (LA 50in50), and design earthquake for soft soil with 10% exceedance in 50 years (LA Soft). The set of Mexican motions (Mexico Soft) was formed of seven narrow banded long duration ground motions recorded in the lake zone of Mexico City. The Mexico Soft motions were scaled up in such way that their peak ground velocity was equal to that corresponding to the EW component of the motion recorded at SCT during 1985. Figures 2 to 4 show spectra for the four sets of motions, and Table 1 summarizes some of the characteristics of the motions included in 'Mexico Soft'. All spectra shown were obtained for elasto-perfectly-plastic behavior and 5% of critical damping.

Figure 2 shows strength spectra for the four sets of motions (S_a stands for pseudo acceleration). The circles identify the location of the corner period (T_s), defined as the period at which the strength spectra decreases after peaking either at a single point or at a plateau. Note that 'LA 10in50' has a corner or dominant period around 0.3 sec, while those of 'LA 50in50', 'LA Soft' and 'Mexico Soft' are around 0.3, 1.0 and 2.0 sec, respectively.

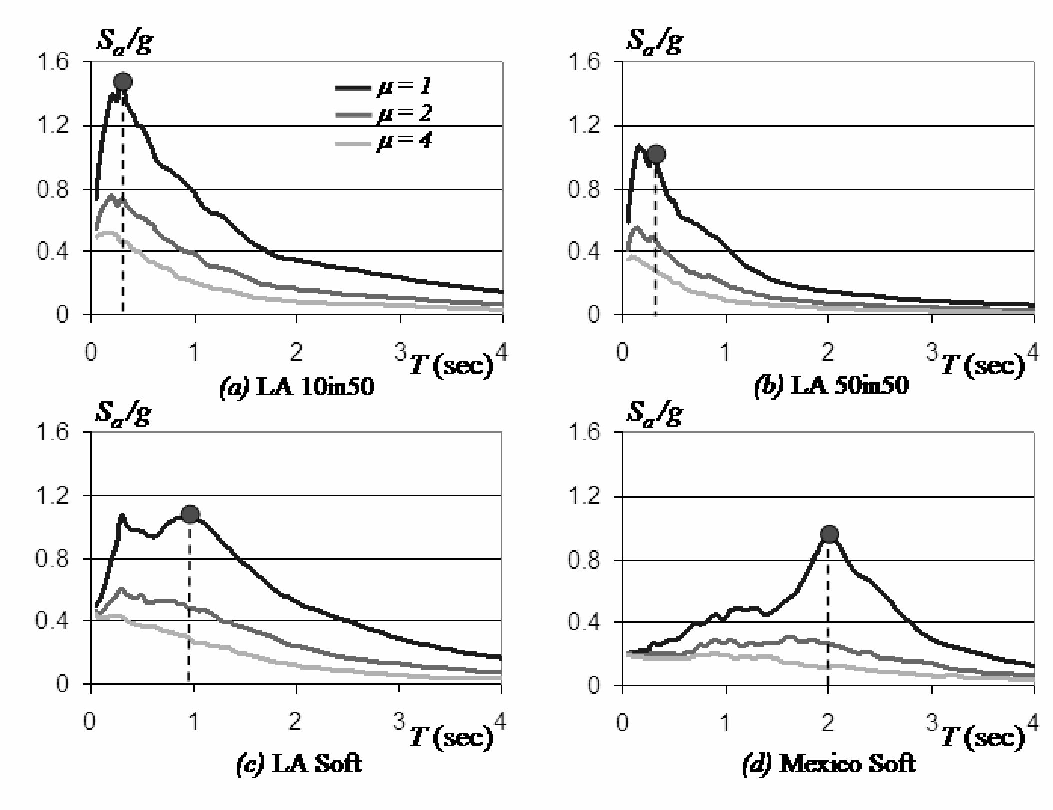


Fig. 2 Strength spectra, 5% critical damping

Figure 3 shows $NE_{H\mu}$ spectra. There is a distinctive feature in the $NE_{H\mu}$ spectra corresponding to the sets of 'LA' motions: starting from very small T, the $NE_{H\mu}$ demand tends to increase until T reaches the value of the corner period, after which it remains fairly constant. For the 'Mexico Soft' set, $NE_{H\mu}$ tends to increase until T reaches the value of the corner period. After that, it tends to gradually decrease with a further increase in T. Note that the corner period defined according to Figure 2 delimits two distinctive zones in the $NE_{H\mu}$ spectra, and that the maximum $NE_{H\mu}$ demands for 'Mexico Soft' are about two to three times larger than those corresponding to the 'LA' motions. For constant ductility, 'LA 10in10' and 'LA Soft' are considered to have low energy content; 'LA 50in50', moderate energy content; and 'Mexico Soft', very large energy content.

Figure 4 shows the coefficient of variation (COV) associated with the mean spectra shown in Figures 2 and 3 for 'LA 50in50' and 'Mexico Soft'. The COV is presented for two purposes: first, to provide an idea of the uncertainty and variability involved in establishing mean spectra; and second, to provide reference values against which the COV associated to the use of the low cycle fatigue model developed herein can be assessed. While the COV of the S_a spectra, corresponding to the four different sets, do not seem to follow a well established pattern; the COV of the $NE_{H\mu}$ spectra do show a surprising similitude for all sets of motion, and is characterized by values usually ranging from 0.4 to 0.6.

SIMPLE DAMAGE MODELS TO PREDICT LOW CYCLE FATIGUE

Although using energy-derived parameters as a representation of repeated cumulative loading allows the formulation of relatively simple seismic design methodologies, this approach should be carefully assessed. The plastic energy dissipating capacity of a reinforced concrete structure does not depend exclusively on its mechanical characteristics, but also on the specifics of its loading history. It has been repeatedly observed that the plastic energy dissipated up to failure by an element or structure can change significantly as a function of the amplitude of the plastic cycles. In particular, the plastic energy dissipated by a large number of small amplitude cycles can significantly exceed that dissipated up to failure through the application of a few large amplitude cycles.

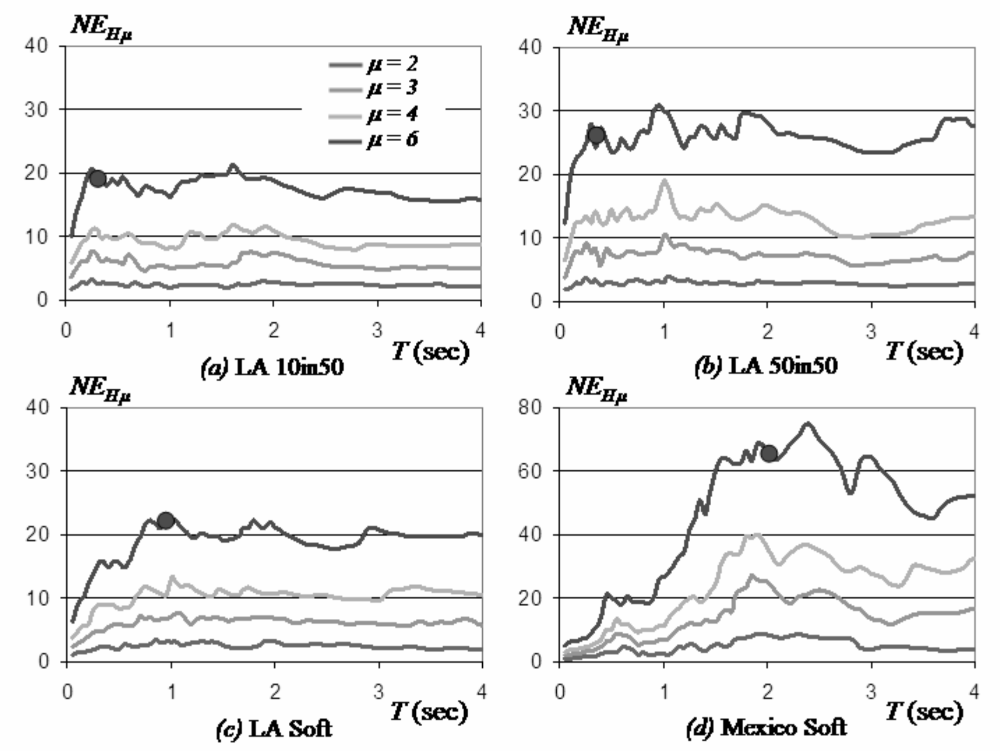


Fig. 3 Normalized plastic energy spectra, 5% critical damping

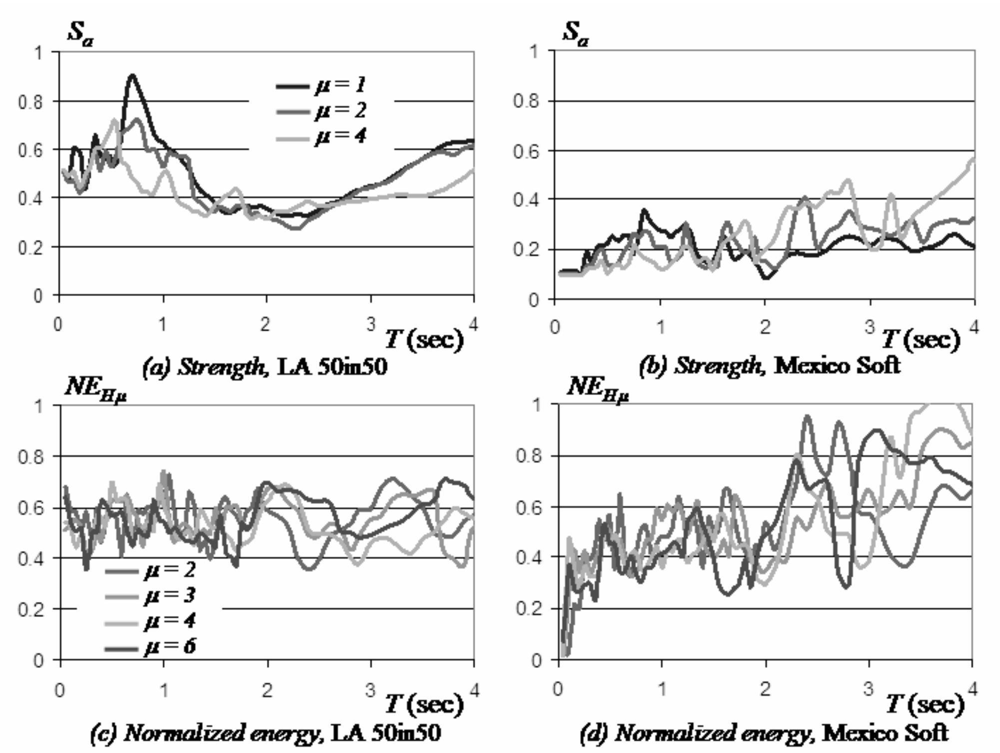


Fig. 4 *COV* of strength and normalized plastic energy spectra, 5% critical damping

Three low cycle fatigue models are discussed next. Two of these models are well-known and have been used extensively to formulate seismic design methodologies that account for low cycle fatigue. The third model is a simple energy-based model introduced by Teran-Gilmore and Jirsa (2004).

1. Park and Ang Damage Index

Park and Ang (1985) have formulated a damage index to estimate the level of damage in reinforced concrete elements and structures subjected to cyclic loading:

$$DMI_{\mathrm{PA}} = \frac{\mu_{\max}}{\mu_u} + \beta \frac{NE_{H\mu}}{\mu_u} \tag{5}$$

where $\mu_{\max}$ is the maximum ductility demand, μ_u is the ultimate ductility, and β is the structural parameter that characterizes the cycling or cumulative deformation capacity of the element or structure (i.e. the stability of its hysteretic behavior). In Equation (5), *DMI* denotes damage index; and the subscript 'PA', Park and Ang. The work done by several researchers suggest that β of 0.15 corresponds to systems that exhibit fairly stable hysteretic behavior; while values of β ranging from 0.2 to 0.4 should be used to assess damage in systems exhibiting substantial strength and stiffness deterioration (Cosenza et al., 1993; Williams and Sexsmith, 1997). Under the presence of repeated cyclic loading into the plastic range, 1.0 represents the threshold value at which low cycle fatigue is expected to occur.

2. Linear Cumulative Damage Theory

A damage index, that can take into account the change in energy dissipating capacity of a structure as a function of its displacement history, can be formulated from the linear cumulative damage theory (Miner's hypothesis). Miner's hypothesis considers that the damage induced by each plastic excursion is independent of the damage produced by any other excursion, in such way that there is a need for a clear convention to define and delimit each excursion. Powell and Allahabadi (1987) suggest that, for earthquake induced deformations, the Rainflow Counting Method is a good option to achieve this.

The linear cumulative damage theory can be formulated as (Cosenza and Manfredi, 1996):

$$DMI_{\mathrm{MH}} = \sum_{i=1}^{Nexc} \left(\frac{\delta_{pi}}{\delta_{ucp}} \right)^b = \sum_{i=1}^{Nexc} \left(\frac{\mu_{pi}}{\mu_{ucp}} \right)^b \tag{6}$$

where *Nexc* is the total number of plastic excursions, δ_{ucp} is the ultimate cyclic plastic displacement, δ_{pi} is the plastic displacement associated to the *i*th excursion, and *b* is the structural parameter that characterizes the cumulative deformation capacity of the structure. Also, $\mu_{pi} = \delta_{pi} / \delta_y$ is the cyclic plastic ductility associated to the *i*th excursion; and $\mu_{ucp} = \delta_{ucp} / \delta_y$, the ultimate cyclic plastic ductility. In Equation (6), *DMI* denotes damage index; and the subscript 'MH', Miner's Hypothesis. DMI_{MH} equal to one implies incipient failure due to low cycle fatigue. Typical values of *b* range from 1.6 to 1.8 (Powell and Allahabadi, 1987). It has been suggested that a *b* of 1.5 is a reasonably conservative value to be used for seismic design and damage analysis of reinforced concrete and steel ductile structures (Krawinkler and Zohrei, 1983; Baik et al., 1988; Cosenza and Manfredi, 1996).

3. A Simple Model to Predict Low Cycle Fatigue

Teran-Gilmore and Jirsa (2004) have recently proposed a simple model to assess the occurrence of low cycle fatigue. Basically, this model represents a simplification of the linear cumulative damage theory through the assumption of a fixed shape for the distribution of the plastic excursions that occur during the ground motion:

$$DMI_{\mathrm{MH}}^{S} = (2-b) \frac{NE_{H\mu}}{\mu_{ucp}} \tag{7}$$

where $NE_{H\mu}$ is the ground motion parameter that quantifies the severity of the plastic demands, μ_{ucp} is the ultimate cyclic plastic ductility, and *b* is the structural parameter that characterizes the cumulative deformation capacity. As before, *b* equal to 1.5 can be considered to be a reasonable conservative value to be used for seismic design of ductile structures. DMI_{MH}^{S} equal to one implies incipient failure due to low cycle fatigue. As suggested by Figure 5, the analytical upper limit for the value of μ_{ucp} is given by

$2(\mu_u - 1)$. In reality, the physical upper limit of μ_{ucp} will be somewhat less than this, because a plastic excursion close to μ_u will damage significantly the capacity of a structure to accommodate plastic deformation in the opposite direction:

$$\mu_{ucp} = 2r\left(\mu_u - 1\right) \tag{8}$$

where r is a reduction factor (less than one). For incipient collapse (DMI^S_{MH} = 1), Equation (7) can be reformulated in terms of μ_u as:

$$NE_{H\mu} = \frac{2r}{(2-b)}(\mu_u - 1) \tag{9}$$

According to Equation (7), the value of $NE_{H\mu}$ estimated from Equation (9) establishes the maximum plastic energy demand that a structure can accommodate before failure due to low cycle fatigue.

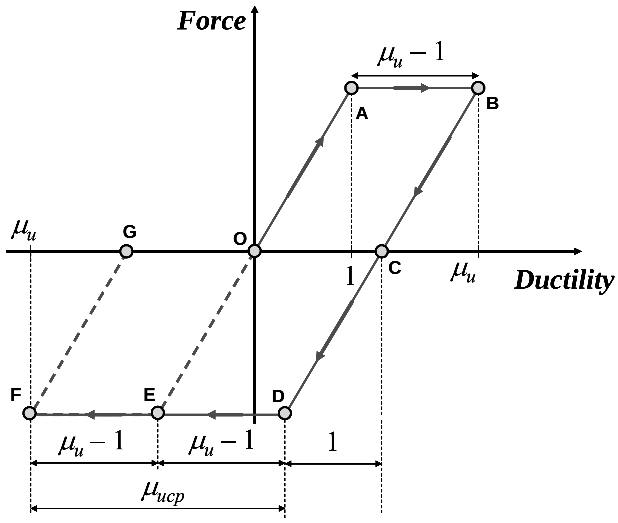


Fig. 5 Upper bound values for the ultimate plastic cyclic ductility

Figure 6(a) shows damage estimates derived from Equations (6) and (7) (b = 1.5 and μ_{ucp} = 7.5) for 'LA 50in50'. The value of μ_{ucp} was established from Equation (8) by assuming μ_u equal to 6 and r equal to 0.75. The discontinuous lines correspond to Equation (6). Equation (7) yields, with respect to Equation (6), higher estimates of damage for $\mu_{\max}$ of 2, slightly higher estimates for $\mu_{\max}$ of 3, and slightly lower estimates for $\mu_{\max}$ of 4.

To explain the results summarized in Figure 6, it is necessary to consider that the energy dissipating capacity of a structure increases as the amplitude of its plastic excursions decreases. In the case of $\mu_{\max}$ of 2, the amplitude of the majority of the plastic excursions $\left(\frac{\mu_{\max}}{\mu_u} = \frac{2}{6} = 0.33\right)$ is small with respect to the ultimate deformation capacity. While Equation (6) accounts for an increased energy dissipation capacity, Equation (7) does not, so that the latter yields higher estimates of damage. As the value of $\mu_{\max}$ increases, the mean amplitude of the plastic excursions increases with respect to the ultimate deformation capacity. Because the energy dissipating capacity of a system will tend to decrease under these circumstances, Equation (7) yields similar and even unsafe estimates of damage with respect to Equation (6) for $\mu_{\max}$ of 3 and 4.

As illustrated in Figure 6(b), the coefficient of variation (COV) of the damage estimates obtained from both equations is practically equal. If the structural parameters involved in Equations (6) and (7) are considered deterministic, the uncertainty in the estimation of the level of damage (Figure 6(b)) is practically equal to that involved in the determination of the energy demands (Figure 4(c)).

Figure 6(c) shows the mean ratio of the damage estimates obtained from Equations (6) and (7) ($DMI^N_{\mathrm{MH}} = DMI^S_{\mathrm{MH}} / DMI_{\mathrm{MH}}$). The ratio shows a strong dependence on $\mu_{\max}$ and a weak variation with respect to T. While the results obtained for 'LA 10in50' and 'LA Soft' are similar to those shown in

Figure 6(c), the ratios corresponding to 'Mexico Soft' are slightly smaller due to its higher energy content. Figure 6(d) shows that the *COV* associated with DMI_{MH}^{N} is very small, fact that implies the damage ratio exhibits practically the same value for all the ground motions included in 'LA 50in50'. Similar results were obtained for the other sets of ground motions.

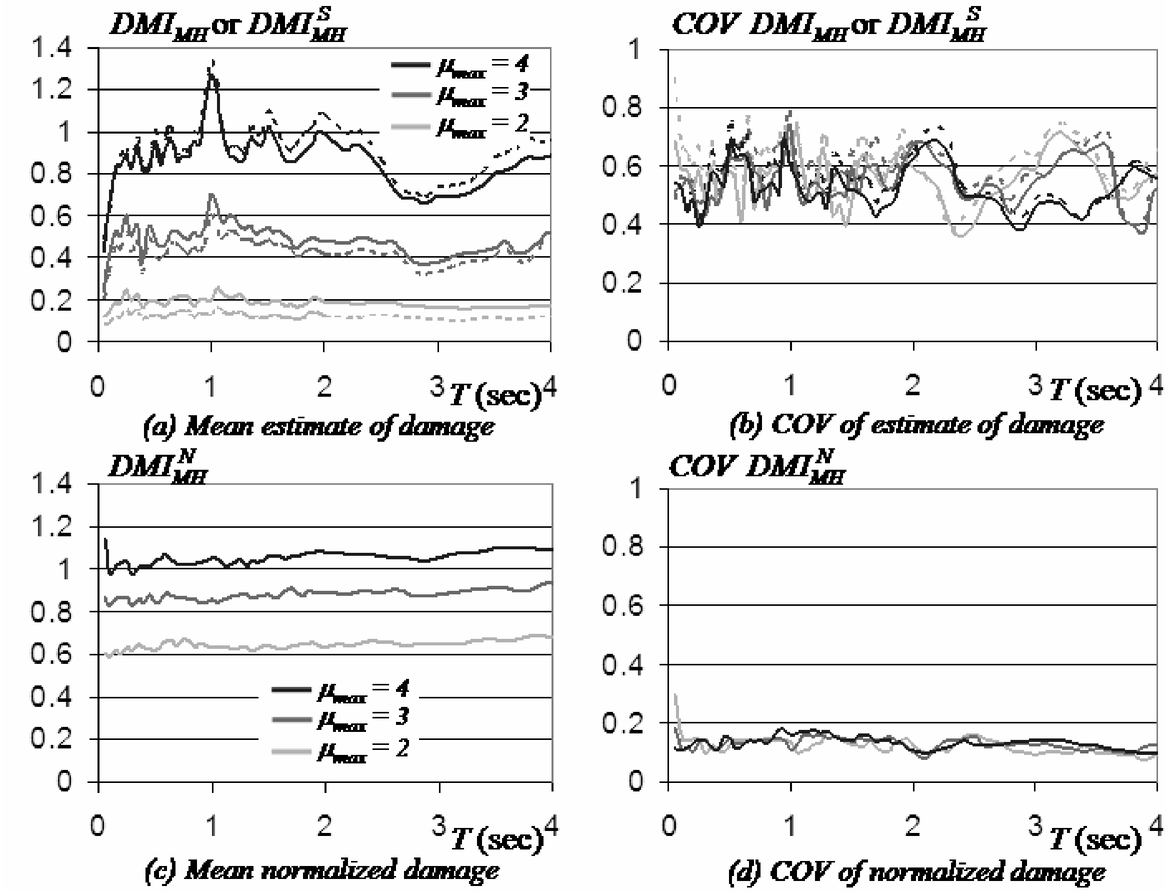


Fig. 6 Estimates of damage from Equation (13), 'LA 50in50'

As the plastic energy demand increases on a given structure, its target ductility should decrease with respect to μ_u, in such way that an increase in the energy content of the ground motion requires the amplitude of the plastic excursions to be reduced relative to the ultimate deformation capacity. Considering the effect of the amplitude of the plastic excursions in the estimates of DMI_{MH}^{S} relative to those of DMI_{MH} (a decrease in this amplitude implies further conservatism in the estimates of DMI_{MH}^{S}), it can be said that Equation (7) yields unsafe assessment of low cycle fatigue when applied to motions with low energy content. As the energy content of the motion goes from low to moderate, the assessment of low cycle fatigue using DMI_{MH}^{S} goes from unsafe to adequate; and, as the energy content goes from moderate to high, this assessment ends up being slightly conservative. As a consequence, the use of DMI_{MH}^{S} to assess incipient failure due to low cycle fatigue yields adequate results for the design of structures subjected to ground motions with moderate and high energy content. In case of structures subjected to low energy demands, DMI_{MH}^{S} yields unsafe assessment of failure; and thus, needs to be complemented with other design criteria.

After extensive studies on the seismic performance of single-degree-of-freedom (SDOF) systems, it was observed that for motions having moderate to very high energy content, Equation (9) with *r* equal to 0.75 yields similar assessment of the occurrence of low cycle fatigue as Equations (5) and (6) (Teran-Gilmore and Jirsa, 2004). Based on this observation, Equation (9) can be rewritten for design purposes as:

$$NE_{H\mu} = \frac{1.5}{(2-b)}(\mu_u - 1) \tag{10}$$

In the case of ductile structures b = 1.5, in such way that:

$$NE_{H\mu} = 3(\mu_u - 1) \tag{11}$$

CUMULATIVE DUCTILITY STRENGTH SPECTRA

The estimation of the lateral strength of a structure within the format of current seismic design methodologies is usually based on the use of constant 'maximum' ductility pseudo-acceleration (strength) spectra. A constant 'maximum' ductility strength spectrum corresponding to ductility (μ) is defined in such way that the pseudo-acceleration (S_a) evaluated at any value of T will result in a lateral strength that is capable of controlling the 'maximum' ductility demand on a SDOF system within a threshold value of μ. Within a practical design procedure that considers the static method of analysis, constant 'maximum' ductility strength spectra can be used as follows:

- Determine the design values of T and μ associated to the structure to be designed. In general, the value of μ is established according to the ultimate ductility capacity (μ_u); and thus, according to the detailing to be used in the structure.
- Evaluate at T the design constant 'maximum' ductility S_a spectrum corresponding to μ.
- Provide the structure with a minimum base shear corresponding to $S_a(T,\mu)W$, where W is the reactive weight of the structure.

1. Concept of 'Cumulative' Ductility Strength Spectra

A constant 'cumulative' ductility strength spectrum corresponding to a 'cumulative' ductility $NE_{H\mu}$ is defined so that its ordinates evaluated at any value of T will result in a lateral strength that is capable of controlling the 'cumulative' ductility demand on a SDOF within a threshold value of $NE_{H\mu}$. As in the case of 'maximum' ductility strength spectra, the ordinates of constant 'cumulative' ductility strength spectra correspond to pseudo-acceleration. Note that although $NE_{H\mu}$ is a normalized plastic energy demand, spectra corresponding to constant $NE_{H\mu}$ have been denoted herein as constant 'cumulative' ductility strength spectra. As pointed out by Equation (4), in the case of elasto-perfectly-plastic systems, $NE_{H\mu}$ is actually equal to the 'cumulative' plastic ductility demand. For systems exhibiting deterioration of their hysteretic behavior, this notation is not strictly correct, but the concept is directly applicable for their seismic design. The term constant 'cumulative' ductility strength spectrum has been used herein to establish a parallelism between the concepts of constant 'maximum' ductility strength spectra and constant 'cumulative' ductility strength spectra.

The use of 'cumulative' ductility strength spectra within the context of the static method of analysis is similar to the current use of strength spectra:

- Determine the design values of T and $NE_{H\mu}$ associated to the structure to be designed. The value of $NE_{H\mu}$ can be established from Equation (10) according to the ultimate and cumulative ductility capacities of the structure (μ_u and b, respectively); and thus, according to the detailing to be used in the structure.
- Evaluate at T the design constant 'cumulative' ductility S_a spectrum corresponding to $NE_{H\mu}$.
- Provide the structure with a minimum base shear corresponding to $S_a(T,NE_{H\mu})W$.

As may be concluded, the design of the lateral strength of a structure using 'cumulative' ductility strength spectra follows the exact same steps currently used in seismic design. The only difference would be that the use of constant 'cumulative' ductility S_a spectrum requires the definition of strength reduction factors that take into consideration the effect of the expected cumulative plastic demands.

2. Use of 'Cumulative' Ductility Strength Spectra in Seismic Design

Within the context of performance-based seismic design for low cycle fatigue, adequate structural performance implies the prevention of failure or collapse of the structure due to excessive plastic deformation demands. To achieve reliable seismic design, it is necessary to provide adequate lateral strength and detailing in such way that the structure can adequately control and accommodate its maximum and cumulative plastic deformation demands. In this paper, it will be assumed that the detailing of the structure is given (i.e., the decision process involved in selecting a particular detailing scheme will not be discussed herein), and that design for low cycle fatigue implies estimating the required lateral strength for the structure to be designed.

Consistent with the format of current seismic design methodologies, it is suggested that strength design for low cycle fatigue be carried out through the use of pseudo-acceleration spectra. From an extensive study on the seismic performance of SDOF systems subjected to ground motions with different frequency content and duration, it has been concluded that the lateral strength to be provided to a ductile structure to avoid failure due to excessive plastic demands should satisfy the following two conditions:

- $\mu_{\max} \leq 0.7\ \mu_u$. First, it is necessary to revise that the 'maximum' ductility demand does not exceed the ultimate deformation capacity of the structure. Note that this condition can be satisfied through the use of constant 'maximum' ductility strength spectra, and that even for motions with low energy content, $\mu_{\max}$ should not be too close to μ_u.
- $NE_{H\mu} \leq \dfrac{1.5}{(2-b)}(\mu_u - 1)$. Second, and particularly for sites that generate long duration motions with narrow-frequency content, it is necessary to revise that the cumulative ductility demand on the structure does not exceed the threshold value given by Equation (10). Note that this condition can be satisfied through the use of constant 'cumulative' ductility strength spectra.

The first condition depends on the ultimate deformation capacity of the structure, which is numerically characterized through μ_u once the detailing of the structure is defined. The second condition depends not only on the ultimate deformation capacity of the structure, but on the stability of its hysteretic cycle, which are numerically characterized through μ_u and b, respectively, once the detailing has been defined. The design strength, to be provided to the structure, should be the larger of the values derived from the two conditions.

Figures 7, 9 and 10 compare 'maximum' and 'cumulative' ductility strength spectra for the different sets of ground motions. Both types of spectra correspond to ductile structures in such way that b = 1.5 and μ_u = 4 and 6. While the 'maximum' ductility strength spectra (black circles) were defined in such way that the value of μ associated to them is equal to $0.7\mu_u$, the 'cumulative' ductility strength spectra (white circles) were defined so that the value of $NE_{H\mu}$ associated to them is equal to $3(\mu_u - 1)$. The figures also include elastic strength spectra (black line), and incipient collapse strength spectra (gray line) derived from the Park and Ang damage index with β of 0.15 (ductile structures). Note that the Park and Ang incipient collapse spectra provide the minimum lateral strength required by SDOF systems so that their level of damage after a ground motion is incipient failure or collapse (DMI_{PA} = 1). Because of the extensive calibration of DMI_{PA} with experimental and field data, the incipient collapse strength spectra derived from it will be used as a benchmark strength design level against which the pertinence of using the two design conditions introduced in this paper will be assessed.

Figure 7 shows strength spectra for the two sets of firm soil motions corresponding to 'LA'. In the case of 'LA 10in50', set formed by motions with low energy content, the ordinates of the constant 'maximum' ductility strength spectra are larger than those of constant 'cumulative' ductility strength spectra, and are very similar to those derived from the Park and Ang damage index. The results shown for 'LA 10in50' suggest that for ductile structures subjected to low energy motions generated in firm soil, seismic design should focus on controlling the maximum ductility demand. Although not shown, similar conclusions can be derived from the results obtained for 'LA Soft', set that corresponds to soft soil motions with low energy content. The similitude of conclusions derived from 'LA 10in50' and 'LA Soft' indicate that in the case of motions with low energy content, seismic design should focus, independently of the type of soil, on controlling the maximum ductility demand.

Figures 7(c) and 7(d) were obtained for 'LA 50in50', set that includes motions with moderate and high energy content. The comparison of results derived from 'LA 10in50' and 'LA 50in50' suggests that as the energy content of the motion is increased, the ordinates of 'cumulative' ductility strength spectra increase relative to those of 'maximum' ductility strength spectra. Nevertheless, the ordinates of 'maximum' ductility strength spectra are still larger than those of 'cumulative' ductility strength spectra, and still very similar to those derived from the Park and Ang damage index. The results obtained for 'LA 50in50' tend to confirm that seismic design of ductile structures in firm soil should focus on controlling their maximum ductility demand.

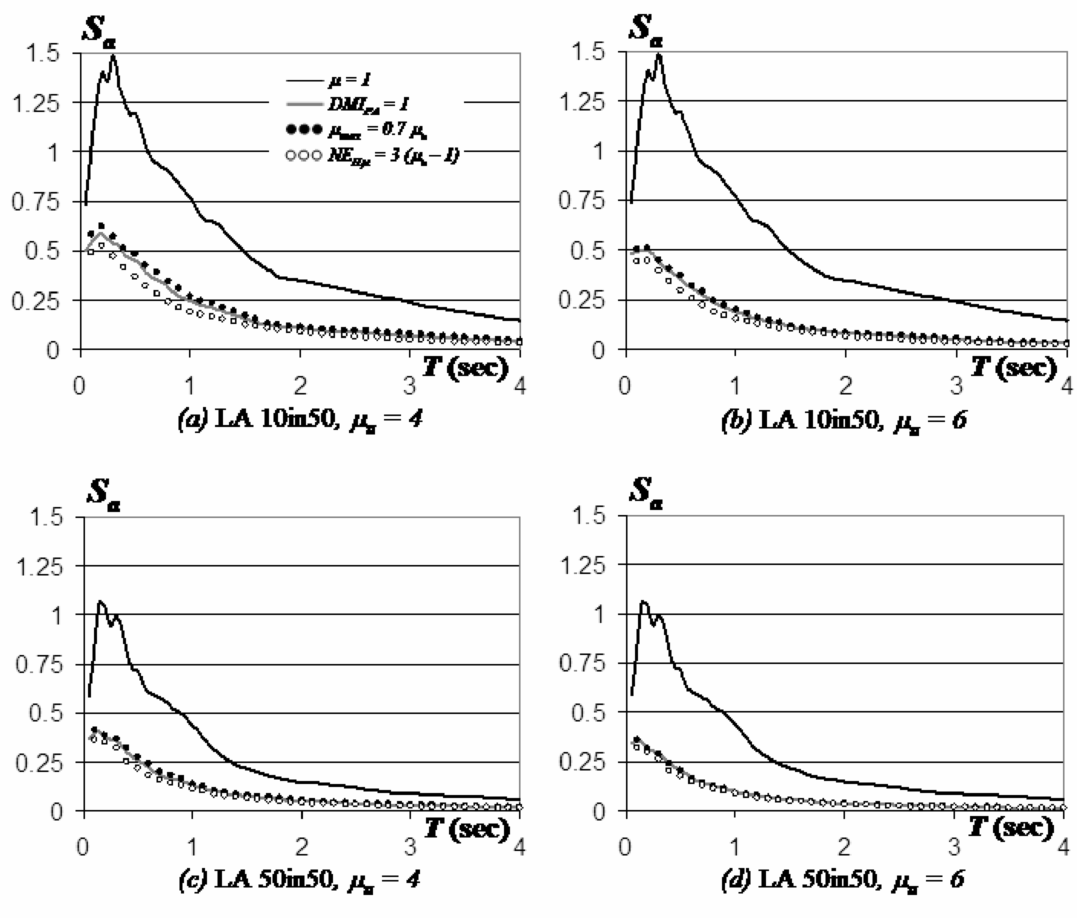


Fig. 7 Strength spectra corresponding to firm soil motions, 5% critical damping

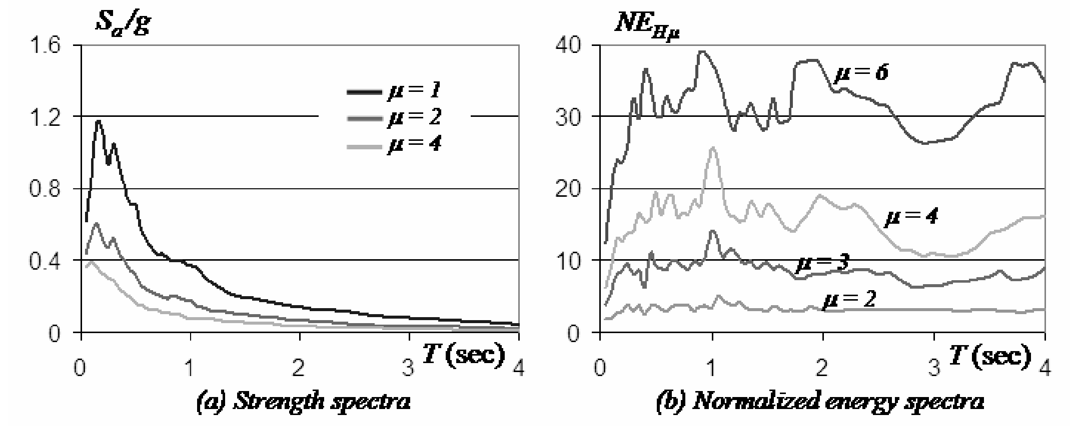


Fig. 8 Spectra corresponding to 'LA MaxEnerg', 5% critical damping

To explore the possibility of low cycle fatigue being an issue for seismic design of structures located in firm soil, a new and fifth set of ground motions was established. The fifth set, denoted as 'LA MaxEnerg', is formed by the twelve motions with the highest energy content in 'LA 50in50'. Figure 8 shows constant 'maximum' ductility S_a and $NE_{H\mu}$ spectra for 'LA MaxEnerg'. After an extensive study of normalized energy spectra corresponding to simulated and recorded firm soil ground motions, it was observed that the energy demands shown in Figure 8(b) are about the largest expected in firm soil.

Figure 9 shows strength spectra for 'LA MaxEnerg'. A further increase in the energy content of the motions from moderate to high, results in a new increase in the ordinates of 'cumulative' ductility strength spectra with respect to those of 'maximum' ductility strength spectra. The increase is so that for 'LA MaxEnerg', both the 'maximum' and 'cumulative' ductility criteria yield similar lateral strength, which in turn is very similar to that derived from the Park and Ang damage index. Although for 'LA

MaxEnerg' the 'cumulative' ductility strength spectra have in some cases slightly larger ordinates than the 'maximum' ductility strength spectra, the small difference in the design lateral strength yielded by both criteria does not seem to justify the consideration of 'cumulative' ductility during seismic design of structures located in firm soils.

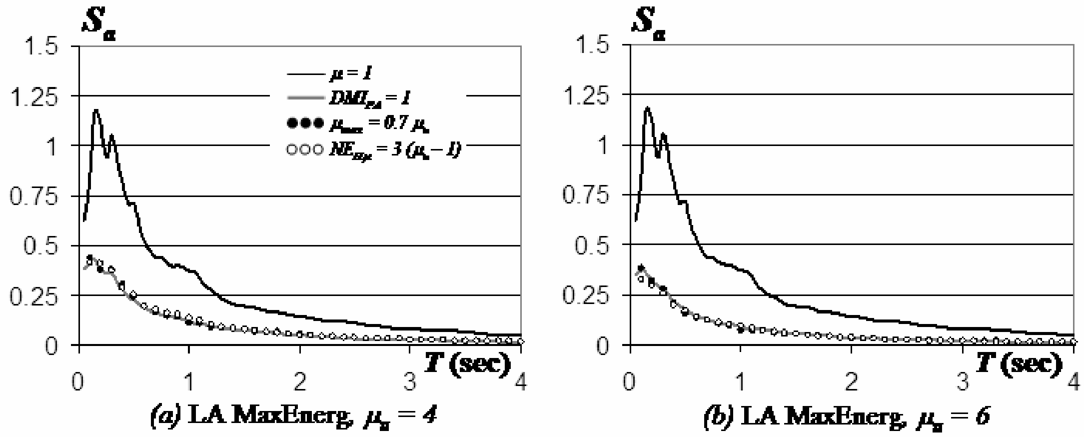


Fig. 9 Strength spectra corresponding to 'LA MaxEnerg', 5% critical damping

Figure 3(d) shows that for 'Mexico Soft', the energy content is very large for systems having T close to 2 sec, value that corresponds to the corner period of this set of motions. In the spectral zone with large energy demands for 'Mexico Soft' (T close to T_s), Figure 10 shows that the 'cumulative' ductility criteria yields considerably larger lateral strength than the 'maximum' ductility criteria. As the value of T departs from T_s, the maximum ductility criteria may yield slightly larger lateral strength requirements, particularly for moderate and small values of T. Note that an envelope of the largest ordinates of both 'maximum' and 'cumulative' ductility strength spectra yields very similar lateral strength than the Park and Ang damage index, except for T close to T_s. Particularly, in this range of T, the 'cumulative' ductility strength spectra yields higher estimates of lateral strength than the Park and Ang damage index. In this respect, recent studies suggest that DMI_{PA} underestimates, with respect to other well-known damage indices, the lateral strength required to prevent low cycle fatigue in SDOF systems having T close to T_s and subjected to motions with narrow frequency content (Teran-Gilmore et al., 2003).

PERSPECTIVES FOR PERFORMANCE-BASED SEISMIC DESIGN

Within the context of current seismic design codes, the design lateral strength is obtained by reducing the design elastic strength spectra evaluated at T by an appropriate strength reduction factor. Because of the need to rationalize the use of strength reduction factors within performance-based design formats, significant research effort has been devoted in recent years to the formulation of transparent and reliable strength reduction factors. The strength reduction factor, R, is defined as:

$$R(\mu, T) = \frac{S_a(1, T)}{S_a(\mu, T)} \tag{12}$$

where $S_a(\mu,T)$ denotes spectral pseudo-acceleration evaluated at μ and T, μ equal to 1 implies elastic behavior, and $S_a(1,T)$ is the seismic coefficient corresponding to the minimum strength that would keep a structure with 5% critical damping in the elastic range.

Equation (12) should be differentiated from strength reduction factors used in current seismic design codes. Normally, strength reduction factors used in practice implicitly consider that the actual lateral strength of a structure can be two to five times its design strength. While Equation (12) only considers reduction in strength due to inelastic behavior, a practical strength reduction factor should account for reductions due to inelastic behavior and expected over-strength in the actual structure.

The value of R strongly depends on μ and T, and is significantly influenced by the type of soil in which the design ground motion is generated. The following trends have been observed for the strength reduction factor corresponding to long duration motions with narrow frequency content (Ordaz and Perez, 1998; Arroyo and Teran-Gilmore, 2003):

- R tends to one as T approaches zero.
- R increases rapidly as the value of T is increased, until it peaks at a value considerably larger than μ at T close to T_s. After it has peaked, R decreases at a high rate until it reaches a value of μ for large T.
- R is not sensitive to the duration of ground motion or other important ground motion characteristics, such as intensity and epicentral distance.
- The values of R corresponding to very soft soil can be affected significantly by a variation in the frequency content of the motion.

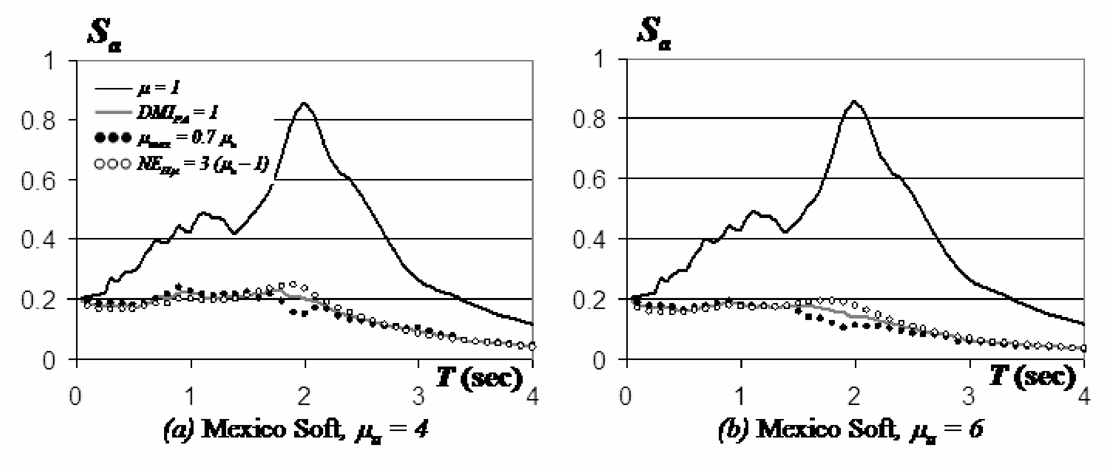


Fig. 10 Strength spectra corresponding to 'Mexico Soft', 5% critical damping

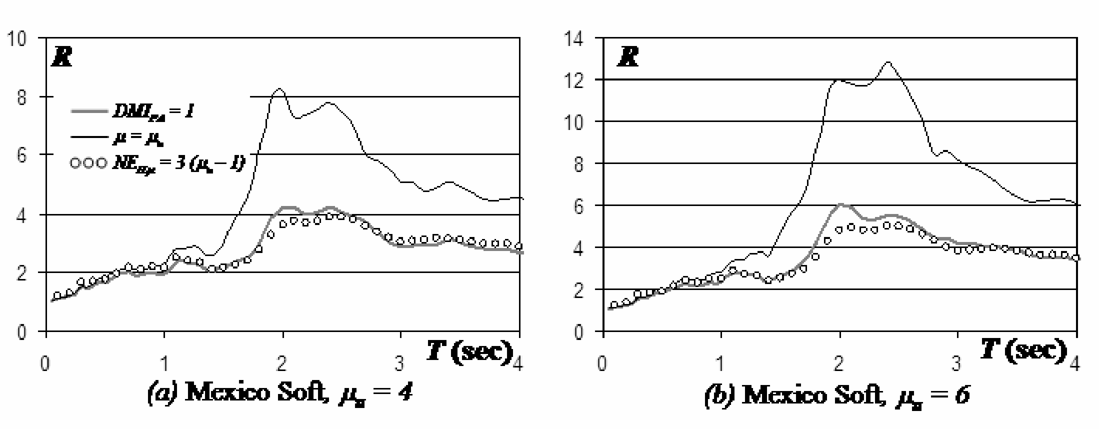


Fig. 11 Strength reduction factors corresponding to 'Mexico Soft', 5% critical damping

Of particular importance to this paper is the observation that for very soft soils, such as those located in the lake zone of Mexico City, R reaches values considerably larger than μ for T close to T_s. This is illustrated in Figure 11 by the continuous black lines, which correspond to values of R for μ equal to μ_u. As shown, under the assumption that the maximum ductility demand undergone by a SDOF system is equal to μ_u, R can reach values up to $2\mu_u$.

The values of R corresponding to constant 'cumulative' ductility strength spectra are defined as:

$$R(NE_{H\mu}, T) = \frac{S_a(\mu = 1, T)}{S_a(NE_{H\mu}, T)} \tag{13}$$

where $S_a(NE_{H\mu}, T)$ denotes spectral pseudo-acceleration corresponding to a 'cumulative' ductility strength spectrum evaluated at $NE_{H\mu}$ and T, and $S_a(\mu = 1, T)$ is the seismic coefficient corresponding to the minimum strength that would keep a structure with 5% critical damping in the elastic range. As shown in Figure 11, the values of R derived for 'Mexico Soft' from the 'cumulative' ductility criteria are very similar to those obtained from the Park and Ang damage index, and considerably smaller than those corresponding to the criteria in which μ is assumed equal to μ_u. In fact, the values of R derived from the 'cumulative' ductility criteria never exceed the value of μ_u, even for T close to T_s.

In the short and medium terms, performance-based seismic design that accounts for the effect of low cycle fatigue should consider the following:

- In the case of very soft soils (long duration motions with narrow frequency content), the design lateral strength should comply with the following two conditions: $\mu_{\max} \leq 0.7\mu_u$ and $NE_{H\mu} \leq \frac{1.5}{(2-b)}(\mu_u - 1)$.

  Consistent with this, the value of R used for design purposes should not exceed the value of μ_u. An option to establish transparently the values of R for practical seismic design is to incorporate the use of constant 'cumulative' ductility strength spectra to current codes. Within this context, 'cumulative' ductility strength spectra may complement or substitute the use of constant 'maximum' ductility strength spectra. In any case, it is important for current codes to allow for rational estimation of the maximum lateral displacement demand in the structure for the purpose of non-structural damage control and avoidance of structural instability.
- In any other type of soil, seismic design should focus on controlling maximum ductility. Nevertheless, the minimum design lateral strength should be such that the maximum ductility demand in the structure is limited to $0.7\mu_u$. Perhaps and based on recommendations made by other researchers (e.g., Panagiotakos and Fardis, 2001), a more stringent limit for $\mu_{\max}$, such as $0.6\mu_u$, can be imposed.

As suggested before, strength reduction factors currently used in practice implicitly consider reductions due to: A) Inelastic behavior; and B) Expected over-strength. The rational use of 'maximum' and 'cumulative' ductility strength spectra should be the basis for the rational and transparent formulation of strength reduction factors for practical performance-based seismic design.

CONCLUSIONS

Damage models that quantify the severity of cumulative loading through plastic energy are simple tools that can be used for practical seismic design. The concept of constant cumulative ductility strength spectra, developed from one such model, is a useful tool for performance-based seismic design.

Seismic design of ductile structures, located in firm soil, should focus on controlling their maximum ductility demand. Nevertheless, even for motions with low energy content, the maximum ductility demand should not be too close to μ_u. The results obtained in this paper suggest that providing earthquake-resistant structures with enough lateral strength to control its maximum ductility demand within the threshold of $0.7\mu_u$ is enough to avoid incipient failure or collapse.

Constant cumulative ductility strength spectra can be used to identify cases in which cumulative plastic demands may become a design issue, and provide quantitative means to estimate the design lateral strength required to avoid failure due to low cycle fatigue. In the case of long duration motions with narrow frequency content, strength requirements should be such that they control adequately the maximum and cumulative ductility demands in the structure according to: $\mu_{\max} \leq 0.7\mu_u$ and $NE_{H\mu} \leq \frac{1.5}{(2-b)}(\mu_u - 1)$, respectively. In this case, the value of R used for design purposes should not exceed the value of μ_u.

Studies are currently being carried out to define if constant 'cumulative' ductility strength spectra should complement or substitute the use of constant 'maximum' ductility strength spectra during seismic design of ductile structures located in the lake zone of Mexico City. As for structures that exhibit irregularities and/or exhibit rapidly deteriorating hysteretic behavior, the set of tools discussed herein become sensitive to the specifics of the local and global hysteretic behavior, and, thus, its application becomes less reliable. While the tools discussed herein can be used to determine the strength and ultimate deformation requirements of ductile structures with stable hysteretic behavior, a more stringent application should be considered for structures with erratic seismic behavior. In this respect, the effects of upper modes and of stiffness and strength degradation in constant 'cumulative' ductility strength spectra should be assessed. Finally, it should be considered that some type of soils, other than those located in the lake zone of Mexico City (e.g., bay mud in the San Francisco Bay area), may exhibit high levels of energy content that may imply the need for using 'cumulative' ductility strength spectra.

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NOTATIONS

The following symbols are used in this paper:

b	=	structural parameter that characterizes the cumulative deformation capacity (stability of the hysteretic cycle) of a structure
DMI_{MH}	=	Miner's Hypothesis (linear cumulative damage theory)
DMI^{N}_{MH}	=	$DMI^{S}_{\mathrm{MH}} / DMI_{\mathrm{MH}}$
DMI^{S}_{MH}	=	Simplified damage model to assess the occurrence of low cycle fatigue
DMI_{PA}	=	Park and Ang damage index
$E_{H\mu}$	=	plastic energy demand
F_y	=	strength at yield
$Nexc$	=	total number of plastic excursions
$NE_{H\mu}$	=	normalized plastic energy, cumulative ductility associated to a constant cumulative ductility strength spectra
PGA	=	peak ground acceleration
PGV	=	peak ground velocity
r	=	reduction factor used to characterize the cyclic deformation capacity of a system
R	=	strength reduction factor
S_a	=	spectral acceleration (pseudo acceleration)
SDOF	=	single-degree-of-freedom
T	=	fundamental period of vibration
T_s	=	corner or dominant period of ground motion
W	=	reactive weight
β	=	constant in Park and Ang damage index that characterizes the cumulative deformation capacity of a reinforced concrete element or structure
δ_c	=	cyclic displacement associated to a plastic excursion
δ_p, δ_{pi}	=	plastic displacement associated to a plastic excursion, subscript indicates ith excursion
δ_{ucp}	=	ultimate cyclic plastic displacement capacity
δ_y	=	displacement at yield
μ	=	maximum ductility demand associated to a constant maximum ductility strength spectra
μ_c	=	cyclic ductility, δ_c/δ_y
$\mu_{\max}$	=	maximum ductility demand
μ_p, μ_{pi}	=	plastic ductility associated to a plastic excursion, δ_p/δ_y and δ_{pi}/δ_y, respectively (subscript indicates ith excursion)
μ_u	=	ultimate ductility, δ_u/δ_y
μ_{ucp}	=	ultimate cyclic plastic ductility, δ_{ucp} / δ_y

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