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THE MAXIMUM MAGNITUDE OF A SEISMIC REGION

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THE MODEL

The maximum magnitude possible in a seismic region will be obtained by introducing a two parameter physical model for the representation of regional seismicity. To obtain the model let us consider that a given portion of earth's crust is crossed by a system of faults of area S, of linear dimension $l=S^{1/2}$.

We assume also that in the system of faults the stress accumulated linearly with time.

$$\varepsilon t \tag{1}$$

where ε is the rate of stress accumulation, that the stress drop $p-p_0$ (p is the stress accumulated and p_0 is the residual stress) is the same for all the earthquakes of the same fault, and that the stress drops to the same value p_0 after all the earthquakes.

The value of t, returning period of the earthquakes of average stress drop $p-p_0$ on a given fault, is given by

$$t=(P-P_0)/\varepsilon \tag{2}$$

Let us represent l and p in a cartesian plane; the extremal values (l_1, l_2) of l and (p_1, p_2) of p define a rectangle; every point of this rectangle; represent a possible Earthquake of our model. It seems acceptabe to assume that $Dl^{-\nu}\, dl$ the number of faults with linear dimensions, in the range $l, 1+dl$, and that the number of the stress drops in the range $p-p_0, p-p_0+dp$ is given by (2) possibly corrected by a factor $P\,(p-p_0)^\alpha$ $(\alpha < 0)$. Then the number of earthquakes in the range, $l, l+dl$, $p-p_0, p-p_0+dp$ and occurred in the time interval T is

$$\frac{TDP}{tl^\nu}\,(p-p_0)^\alpha\, dl\, dp \tag{3}$$

where t is the return period of the earthquakes with stress drop $p-p_0$. It can be proved mathematically that the form here assumed for the distribution function of l and $p-p_0$ is the only compatible with observations.

Depending on the values of l_1, l_2, p_1, p_2, we can have two patterns for the paths of integration in relation to the crossing of these curves with the lines $p=p_1$ and $p=p_2$. The most realistic situation is that of Fig. 1.

A direct integration of formula (3) gives the cumulative distribution functions $n\,(W)$; from it one may then obtain the frequency distribution function $\bar{n}\,(W)=-(dn/dW)$. W is the strain energy released by the earthquake related to l and p by

$$W=kl^3\,\frac{p^2-p_0^2}{2\mu} \tag{4}$$

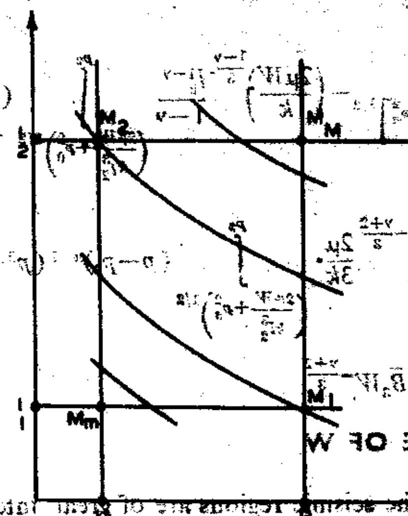

Fig. 1

where k is a geometric factor. In turn W is related to the magnitude M by

$$W=10^{\beta+\gamma M} \tag{5}$$

where η is the seismic efficiency and $\beta=12$ $\gamma=1.5$ are world average values. They have different expressions depending on the values

$$W_1=\frac{kl_1^3(p_2^2-p_0^2)}{2\mu} \qquad W_2=\frac{kl_2(p_1^2-p_0^2)}{2\mu} \tag{6}$$

and on the maximum and minimum values of W. When $W_1 < W_2$ these values, called corner energy define the interval in which $\log \bar{n}(W)$ is linear. A direct integration of (10) when $W_1 < W_2$ for $W_1 < W < W_2$ we have

$$n=\frac{TDP_2}{1-\gamma}\left\{ l_2^{1+\nu}\,\frac{(p_2-p_0)^{\alpha}-(p_1-p_0)^{\alpha}}{\alpha}-\left(\frac{2W\mu}{k}\right)^{\frac{1-\nu}{3}}\int_{p_1}^{p_2}(p-p_0)^{\alpha-1}(p^2-p_0^2)\,dp\right\}$$

$$\log \bar{n}=\log\left\{TPD_2\left(\frac{2\mu}{k}\right)^{-\frac{\nu-1}{3}}\int_{p_1}^{p_2}(p-p_0)^{\alpha-1}\,(p^2-p_0^2)^{\frac{\nu-1}{3}}\,dp\right\}$$

$$-\frac{\nu+2}{3}\log W=A_2-\frac{\nu+2}{3}\log W$$

For $W_2 \leq W < W_M$ we have

$$n = \frac{TPD\epsilon}{1-\nu}\left\{\left[\frac{(p-p_0)^{\alpha}}{\alpha}\right]_{\left[\frac{2\mu W}{kl_2^2}+p_0^2\right]^{1/2}}^{p_2} - \left(\frac{2\mu W}{k}\right)^{\frac{1-\nu}{2}} \frac{l_2^{1-\nu}}{1-\nu} \int_{\left(\frac{2\mu W}{kl_2^2}+p_0^2\right)^{1/2}}^{p_2} (p-p_0)^{\alpha-1}(p^2-p_0^2)^{\frac{\nu-1}{2}} dp\right\} \quad \ldots(8)$$

$$\bar{n} = \frac{PTD\epsilon}{\nu-1}\left(\frac{2\mu W}{k}\right)^{-\frac{\nu+2}{3}} \frac{2\mu}{3k} \int_{\left(\frac{2\mu W}{kl_2^2}+p_0^2\right)^{1/2}}^{p_2} (p-p_0)^{\alpha-1}(p^2-p_0^2)^{\frac{\nu-1}{2}} dp$$

$$= -\bar{A}_3 W^{-1+\frac{\alpha}{2}} + \bar{B}_3 W^{-\frac{\nu+2}{3}}$$

THE MAXIMUM VALUE OF W

The value of W_M of the seismic regions are of great interest in many problems, they can be obtained by fitting formulae (8) to the catalogue of earthquakes of that region.

During the faulting, the fault may overshoot the equilibrium position and build a stress of sign opposite to that which generated the earthquake, as in the case of reverse faulting; it is therefore reasonable to assume the average value $p_0=0$.

To obtain the maximum value of W, first we obtain ν from the linear part of $\bar{n}(W)$ and α from the distribution of stress drop, then with the available values of $\bar{n}(W)$ for $W \geq W_2$ we obtain the parameters $\bar{A}_3, \bar{B}_3$.

Finally the adjusted values of $\bar{A}_3$, $\bar{B}_3$ give

$$\log W_M = \left(\frac{1-\nu}{2} - \frac{\alpha}{2}\right)^{-1} \log \frac{\bar{A}_3}{\bar{B}_3} \quad \ldots(9)$$

For practical purpose formula (8) and (9) can be written for $p_0=0$ which is a reasonable assumption

$$\bar{n} = -\bar{A}_3' \, 10^{\frac{\alpha\gamma}{2}M} + \bar{B}_3' \, 10^{\frac{1-\nu}{3}\gamma M}$$

$$M_M = \left(\frac{\nu-1}{3}\gamma + \frac{\alpha\gamma}{2}\right)^{-1} \log \frac{\bar{B}_3'}{\bar{A}_3'}$$

$$\bar{n} = \frac{TDP\epsilon \, l_n 10}{3}\left[p_2^{\alpha+\frac{2}{3}(\nu-1)}\left(\frac{2\mu 10^{\beta}}{\eta\kappa}\right)^{\frac{1-\nu}{3}} 10^{\frac{1-\nu}{3}\gamma M} - l_2^{-\frac{3}{2}\alpha-\nu+1}\left(\frac{2\mu 10^{\beta}}{\eta\kappa}\right)^{\alpha/2} 10^{\frac{\alpha\gamma}{2}M}\right]$$

$\bar{A}_3'$ and $\bar{B}_3'$ should be used as best fit parameters for the data with $M > M_2$ $(W > W_2)$; $\frac{1-\nu}{3}\gamma = b$ should be computed in the range $M_1 < M \leq M_2$ $(W_1 < W < W_2)$ as it was done by Caputo Console (1977), as well as γ and α.

PRECURSORS OF EARTHQUAKES

The analitic study of the parameter $\bar{A}_2$ of (8) suggests that an accurate analysis of the catalogues of earthquakes may allow to detect precursors of earthquakes. In fact differentiating (8) with respect to p_2 we obtain.

$$\left\{\frac{\partial \bar{A}_2}{\partial p}\right\} dp_2 = \frac{\left(\frac{2}{3}(\nu-1)+\alpha\right) p_1^{2/3(\nu-1)+\alpha}}{p_2^{2/3(\nu-1)+\alpha} - p_1^{2/3(\nu-1)+\alpha}} \frac{dp_2}{\ln 10} \qquad \ldots(10)$$

since $p_1 < p_2$ and $2(\nu-1)+3\alpha > 0$ we may write,

$$\frac{\partial \bar{A}_2}{\partial p_2} dp_2 = \left(\frac{2}{3}\ (\nu-1)+\alpha\right) \frac{dp_2}{p_2 \ln 10} \qquad \ldots(11)$$

and for the values of α and γ obtained for California we have

$$d\bar{A}_2 = 0.2\,\frac{dp_2}{p_2} \qquad \ldots(12)$$

Therefore a variation of the regional values of p_2 due to dry or wet dilatancy, or to an additional load of water in case of a dam or to atmospheric pressure changes, may cause a variation in the regional number of earthquakes per unit time by a factor

$$\bar{c} = 10^{0.2 \frac{dp_2}{p_2}} = 1 + 0.5 \frac{dp_2}{p_2} \qquad \ldots(13)$$

and we may see that a variation of 40% in p_2 could cause a variation of 20% in the number of earthquakes per unit time.

In case of an additional load of water in an artificial lake the factor $\bar{c}$ may be of several units or tens. The same could be true for tectonic causes.

Another relevant precursor could result from the variation of M_1 caused by a variation in the regional condition of stress. In fact we have from the first formula of (6) and (5) differentiating with respect to p_2.

Since for $M = M_1$ the derivative of the function $\log \bar{n}$ has a conspicuous discontinuity and the curve forms a cusp, the variation of the position of this cusp caused by variation of p_2 should be detectable. A change of 40% in p_2 should cause a change of 20% in the number of earthquakes per unit time and also M_2 should change of 0.5.

The analysis of a catalogue of the earthquakes of a region of Pakistan (assembled by L. Seeber of Columbia University) suggests that $M_1 < 0$; the number of earthquakes per unit time with $M \simeq 0$ to consider is therefore 10^4. In the most active regions this number of earthquakes is sufficiently large to allow to observe fluctuations of M_1 with reasonable accuracy and in relatively short time. If the value of M_1 is too small and we cannot have a catalogue complete enough to be able to detect it, the value of M for which the function $\log \bar{n}$ ceases to be linear, in the range of small values of M, could serve for the same use as M_1.

Another obvious precursor could be originated by a change in ε. In fact from (8) differentiating with respect to ε we obtain

$$\frac{\partial \bar{A}_2}{\partial \varepsilon} d\varepsilon = \frac{\partial \varepsilon}{\varepsilon \ln 10} \qquad (15)$$

The variation of $\bar{A}_2$ over the centuries observed in various parts of the world (e.g. Anatolia and Italy) could be related to the variation of ε and from the variation of $\bar{A}_2$ one could retrieve the variation of ε. For that we are concerned here, a variation of 20% in ε causes an equal variation in the number of earthquakes per unit time.

Finally we must note that a variation in the local condition of stress near the surface of the earth, due to natural causes or to human activity, can trigger a new set of faults near the surface. In this set of faults the small faults would be more numerous, because at the surface the confining pressure is smaller. This would cause an increase in the value of ν which in turn would cause a decrease in the value of $\bar{B}_2$ has it has been observed in the regions where large loads of waters have been applied to the surface of the earth.

CONCLUSIONS

The model presented here to represent the statistical data on earthquakes which are available today fits satisfactorly the data of most seismic regions of the world. If a catalogue of the magnitudes of a seismic region is available, the formulae enable to estimate the maximum magnitude possible in the region, also the model allows to estimate the density distribution function of the faults and of the stress drops of the region as function of their size.

Moreover the model suggests some direct links between physical phenomena occurring in the crust of the Earth and observable precursors of earthquakes.

Analogous formulae have been obtained for the seismic moment. However since the seismic moment is rarely measured we shall not report the formulae here; we shall simply note that the check on all formulae made with available data are successful.

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A NOTE ON THE UNCERTAINTY IN GEOTECHNICAL SCIENCES

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The branches of human learning such as 'Geosciences' or 'Geomechanics' or 'Earthsciences' are developing fast at present, and the rate of development has been very fast especially in last two or three decades. It has left a remarkable impact on other sciences such as astrophysics, astronomy, heavy constructions, planetary formation etc. When epistemologists try to measure the amount of progress in these branches compared to other branches of learning such as medical, agricultural and physical sciences it is observed that the progress made in Earthsciences is less as compared to the other branches. It is also further observed that the earth sciences have yet to acquire the desired degree of maturity and advancement. The common man thinks of 'Science' in terms of physical sciences and to emphasise this point it is often quoted that the off shoots of Physics such as Electronics and Mechanical engineering, off shoots of chemistry such as Chemical technologies of fuel, pharmaceuticals etc. have developed very fast. Finally, to justify these comparisons and the arguments, it is quite common to hear that there is no Nobel Prize winner in the field of geosciences. However, it is conveniently forgotten that the Nobel prize is primarily intended to be awarded for the contributions in the field of Pure Physics, Chemistry, Medicine and Peace.

The relative progress of physical sciences and geosciences is discussed in the following pages from logical and developmental point of view. Till the end of last century, most of the scientific activities were grouped as in the classical term 'Natural Philosophy' and for the last few decades Physics and Chemistry have been creating various frontiers of knowledge and it has undergone tremendous progress with innumerable discoveries which have not only been beneficial to these subjects but to other subjects also. A few examples can be sighted to illustrate the case. The discovery of transister was in the field of pure physics, however, its applications have been made to several fields, especially in electronics so much that a completely new branch of solid state Physics has been established. Similarly, the discovery of Laser has been used in the fields of optical surgery, radar communication, welding technology etc. These discoveries have been of greater use in human activities and are therefore considered as 'Big discoveries'. In the fields of Earth Sciences also there have been very important discoveries such as 'Liquid core' of the earth, Mohorovicic discontinuity, the free oscillations of the earth (after the great Chilean earthquake of 22 may 1960) etc. But the common man is not so much aware of these discoveries as he is aware of the discoveries in the physical sciences.

When a common man sees a big dam, tall building, long tunnel, high transmission towers, transoceanic canals like Suez or Panama, Zuider sea reclamation in Netherlands, or Beas-Sutlej Link Project in India etc. he rarely gets an idea about the lion's share of Earth Scientist in these constructions. The foundation investigations and other geological and geotechnical investigations are of prime importance in these activities. In recent times there is one example where the earth science advancement has been useful is that of Berkeley Nuclear reactor. The design and the other details were completely ready, but when it was found that it lies on an active geological fault, the site was changed.

The main reason for the lack of understanding on the part of common man is

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that there are very few 'Popular' type of writings in the subject and secondly, most of the results, findings and observations in this field have been put as 'Tentative', 'varies between', 'is of the order of', 'lies n the range of' or 'likely to be of the order of', 'probable values lie in the range of, etc. and therefore these values are always taken as probabilistic or undeterministic in nature. A modern Physicist or an Engineer who is more or less bent on knowing the 'exact' values believes it as not so correct though he believes it to be 'of the order of' with certain statistical distribution like the Gaussian, Normal etc. There is a peculiar characteristics with the design engineer, even though he accepts the value of a particular geotechnical parameter say as 8.5 or 11.75 with certain reservations, for his design purposes he takes the parameter as 10 only. It is left to the readers whether this decision to take the value as 10 is due to inherent doubt in the geotechnical measurements or due to lack of confidence in the design engineer himself.

The Statistical Laws in Physical and Geotechnical Sciences:

It is really very much interesting to note that the same statistical laws of distribution have different degrees of acceptances in various branches of knowledge. When a medical researcher studies effect of some chemical on the functioning of a particular organ of human body, on a small section of population in a particular locality, his findings are readily accepted by the majority of population without any reservations. However, if a geoscientist makes certain claims about the *P* velocities in a layered formation, using the same statistical laws, it is not readily accepted. This is due to lack of proper interaction between the subject and the common man. It should be always borne in mind that the statistics does not help in understanding the subject but it is useful in understanding the observed facts with certain laws of distribution, occurrence, non-occurrence and the failures. In all the measurements, the factor of uncertainty is always inherently associated with certain degree of failures or tolerance. The amount of uncertainty may vary depending upon the type of measurement and the method of measurement itself, this is the main point where the earth scientists are unable to compete with the physical scientist. However, the element of uncertainty in geosciences should not be taken as the conventional uncertainty but it should be taken as the normalcy. As in Physics the physical laws of measurements are always governed by the Heisenberg's principle of uncertainty though the amount of uncertainty is always very small as compared to the conventional measurements but it is not so small as compared to the atomic dimensions. But this is usually forgotten and the physical measurements are always taken as 'Exact', not only by the common man but by the scientists themselves. When a geoscientist says that the Indian continent is moving with a velocity of 3 to 5 cms per year in the northeastern direction, this statement should be equally accepted as a particular radioactive substance is likely to emit 10^{13} radiations per second. In a radioactive substance if there are N_0 number of nucleii then it is said that at any time t, N_t number of nucleii will disintegrate and various radioactive constants are purely based on statistical calculations. However, all the decay constants are normally taken as 'Exact'. When a geomorphologist says that the life of reservoir is likely to be of the order of 250 to 300 years this statement is taken not so seriously and at times it is ridiculed as vague.

It is, therefore, much partinent to note that all the branches of science are always having some bias of uncertainty' in all the findings and unfortunately this important factor is not considered in its proper seriousness. Mathematics which is known as queen of sciences is also affectd by this factor. The Mathematicians have been claiming their subject as one of the perfect sciences. It has its fundamental constants like 'e' and 'π' and several formulae have been developed on these basis, however, these constants are incommensurable and the exact values of these constants are not known, not only this the mathematicians themselves have proved that the exact values of these constants cannot be found out. Same is the case with astrophysical measurements, the values of various parameters like distance, diameter, velocity, temperature of a celestial objects are always associated with certain amount of uncertainty.

It can therefore be said that all the measurements in various branches of learning are invariably associated with certain amount of uncertainty as explicit characteristics of the findings itself, and measurements cannot be segregated as 'perfect', 'exact', 'absolute', 'free of error' etc. The results are always to be judged within the limits of the tolerance. This law is equally applicable to the geotechnical sciences, and the findings in this branch should be seen with understanding the above principle for the harmonious interaction between various branches of learnings.