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SINGLE DEGREE FREEDOM SYSTEM WITH AN ASSEMBLAGE OF VIBRATION ABSORBERS UNDER SINUSOIDAL EXCITATIONS

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Synopsis

Vibration absorbers have so far been used with machines and machine foundations subjected to sinusoidal excitations. In such cases a single absorber is required to reduce the large amplitude of vibration of primary system at resonance. In this paper the influence of providing multiple vibration absorbers has been presented. Experimental and analytical investigations were carried out for a single degree freedom system with an assemblage of vibration absorbers. The two results, analytical and experimental, compare very well with each other.

Introduction

The influence of single absorber on the response of basic single degree freedom system has been studied by several authors. To find the effect of multiple vibration absorbers in reducing the response of primary system, experimental and analytical investigations have now been carried out. Sinusoidal excitations were chosen for the test since these can be easily produced in laboratory with the help of mechanical oscillator. For the experiment, portal frame was selected as the basic system as it is a simplified type of framed structure and can be approximated as single degree freedom system. Steel was chosen as the material for frame because of its linear properties over a large range. Absorbers were also made of steel leaf springs.

Free and forced vibration response of frame with and without absorbers was recorded and response curves were drawn. Several combinations of absorbers with the basic system were tried. Analytically the response was determined by writing equations of motion and solving them with the help of numerical techniques. The two results were well in accordance with each other.

An assemblage of vibration absorbers is effective not only at one exciting frequency but at several other frequencies also. The response of parent system further decreased with the increase in number of absorbers.

The Model

A model of single bay portal frame made of steel (as shown in fig. 1) was fabricated. It was mounted on a shaking table, which could have horizontal motion in one

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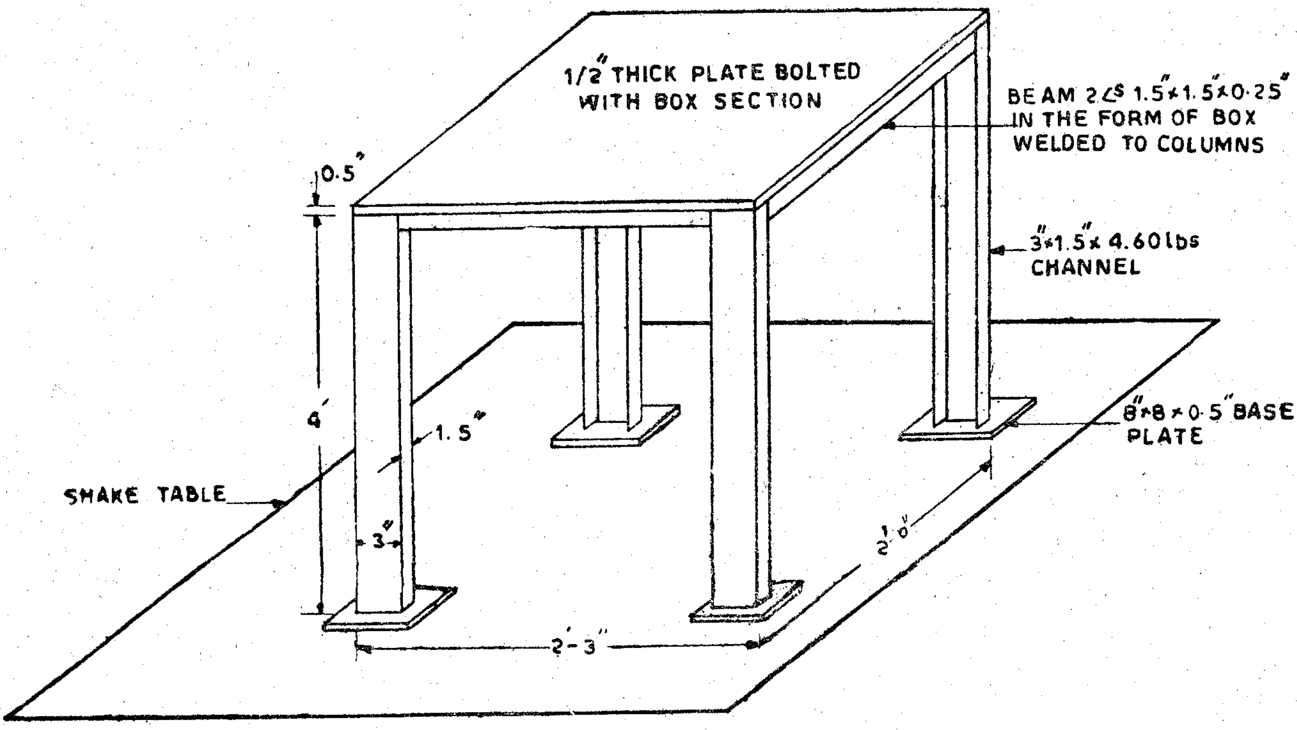

Fig. 1. Model of Portal Frame Mounted on Shake Table

direction. Vibration absorbers were made of steel leaf springs with concentrated mass at one end as shown in fig. 2.

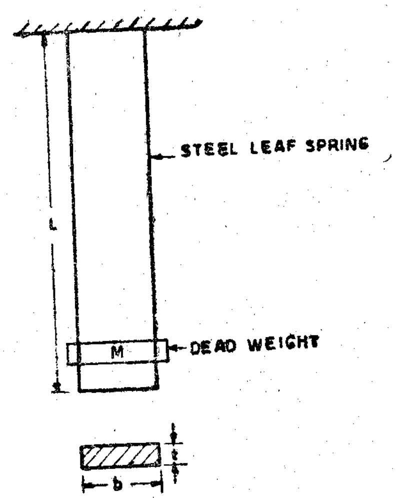

Fig. 2, Vibration Absorber Made of Steel Leaf Spring

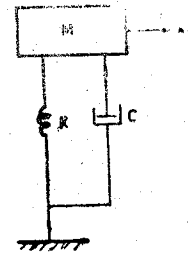

Fig. 3. Equivalent SDF Model

Experimental Setup

The model alone as well as with several combinations of absorbers, was excited on a shake table. The shake table was given motion by a mechanical oscillator driven by variable speed D C. motor. The oscillator works on the principle of two eccentric masses rotated in two opposite directions. The centrifugal forces get added up in one direction and get cancelled in the other perpendicular direction. Thus it gives vibratory force either in horizontal direction or in vertical direction depending upon the mounting position of oscillator.

The accelerations of table, frame and absorbers were picked up by acceleration pickups The pickup was connected to a bridge circuit and the output of bridge was amplified by a Universal amplifier. The amplified signal output was recorded on an ink writing oscillograph. An experimental run consists of measuring the accelerations for various exciting frequencies.

Free Vibration Behaviour of Frame and Absorbers

The frame as well as absorbers can be represented by an equivalent single degree freedom system as shown in fig. 3.

Free vibration record of the system was taken after giving initial displacement to the mass Record was taken with the help of a pickup, amplifier and ink writing oscillograph. The following frequencies and damping properties of frame and absorbers were found from free vibration records.

TABLE 1

System	Frequency c/s	Percentage of Critical Damping	Equivalent lumped mass lbs.
Frame	8.58	0.625	180.00
Abs. 1	8.60	1.272	27.50
Abs. 2	9.25	1.200	27.50
Abs. 3	9.92	1.110	27.50
Abs. 4	10.30	1.070	27.50

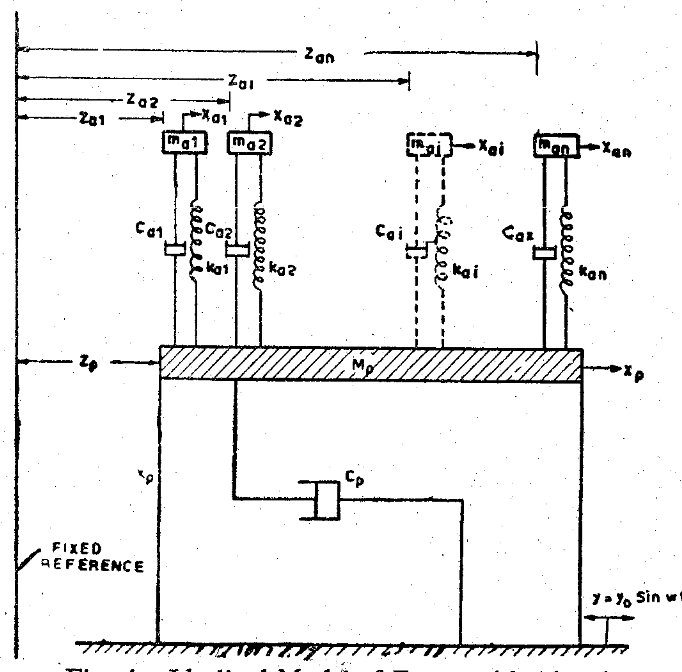

Fig. 4. Idealised Model of Frame with Absorbers

Response of Model to Sinusoidal Excitations

(A) Experiment

The accelerations of table base, frame and absorbers were measured for various exciting frequencies Several runs were made with different combinations of vibration absorbers with the parent system. Response curves for acceleration ratio versus frequency ratio were plotted. Acceleration ratio is the acceleration of frame or of absorbers divided by acceleration of table at any time and frequency ratio is the exciting frequency divided by natural frequency of frame alone.

(B) Analysis

The mathematical model of frame with absorbers is shown in fig. 4. The equations of motion for this model can be written as follows :

$$m_p \ddot{x}_p + (c_p+c_{a1}+c_{a2}+\dots c_{an})(\dot{x}_p-\dot{y}) - c_{a1}(\dot{x}_{a1}-\dot{y}) - c_{a2}(\dot{x}_{a2}-\dot{y}) \dots c_{an}(\dot{x}_{an}-\dot{y})$$
$$+ (k_p+k_{a1}+k_{a2}+\dots k_{an})(x_p-y) - k_{a1}(x_{a1}-y) - k_{a2}(x_{a2}-y) \dots k_{an}(x_{an}-y) = 0$$

$$m_{a1}\ddot{x}_{a1} - c_{a1}(\dot{x}_p-\dot{y}) + c_{a1}(\dot{x}_{a1}-\dot{y}) - k_{a1}(x_p-y) + k_{a1}(x_{a1}-y) = 0$$

$$m_{a2}\ddot{x}_{a2} - c_{a2}(\dot{x}_p-\dot{y}) + c_{a2}(\dot{x}_{a2}-\dot{y}) - k_{a2}(x_p-y) + k_{a2}(x_{a2}-y) = 0$$

$$\dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots$$

$$\dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots$$

$$m_{an}\ddot{x}_{an} - c_{an}(\dot{x}_p-\dot{y}) + c_{an}(\dot{x}_{an}-\dot{y}) - k_{an}(x_p-y) + k_{an}(x_{an}-y) = 0 \qquad (1)$$

where,

m_p — lumped mass of parent system.
c_p — damping constant for parent system.
k_p — stiffness of parent system.
m_a — lumped mass of absorber.
c_a — damping constant for absorber.
k_a — stiffness of absorber.
y — base motion.
x_p — displacement of mass m_p.
x_a — displacement of mass m_a.

Equation 1 can be simplified and written in the matrix form as follows :

$$\{\ddot{Z}\} + [D]\ \{\dot{Z}\} + [S]\ \{Z\} = -\ \{\ddot{y}\} \tag{2}$$

where,

$$[D] = 4\pi \begin{bmatrix} \left(\frac{\zeta_p}{T_p} + \frac{a_{a1}\zeta_{a1}}{T_{a1}} + \frac{a_{a2}\zeta_{a2}}{T_{a2}} + \dots + \frac{a_{an}\zeta_{an}}{T_{an}}\right) & -\frac{a_{a1}\zeta_{a1}}{T_{a1}} & -\frac{a_{a1}\zeta_{a2}}{T_{a2}} & \dots & \frac{a_{an}\zeta_{an}}{T_{an}} \\ -\frac{\zeta_{a1}}{T_{a1}} & \frac{\zeta_{a1}}{T_{a1}} & & & \bigcirc \\ -\frac{\zeta_{a2}}{T_{a2}} & & \frac{\zeta_{a2}}{T_{a2}} & & \\ \vdots & & & \vdots & \\ -\frac{\zeta_{an}}{T_{an}} & \bigcirc & & & \frac{\zeta_{an}}{T_{an}} \end{bmatrix}$$

$$[S] = 4\pi^2 \begin{bmatrix} \left(\frac{1}{T_p^2} + \frac{a_{a1}}{T_{a1}^2} + \frac{a_{a2}}{T_{a2}^2} + \dots + \frac{a_{an}}{T_{an}^2}\right) & -\frac{a_{a1}}{T_{a1}^2} & -\frac{a_{a2}}{T_{a2}^2} & \dots & -\frac{a_{an}}{T_{an}^2} \\ -\frac{1}{T_{a1}^2} & \frac{1}{T_{a1}^2} & & & \bigcirc \\ -\frac{1}{T_{a2}^2} & & \frac{1}{T_{a2}^2} & & \\ \vdots & & \bigcirc & \vdots & \\ -\frac{1}{T_{an}^2} & & & & \frac{1}{T_{an}^2} \end{bmatrix}$$

$$\{Z\} = \begin{Bmatrix} x_p - y \\ x_{a1} - y \\ x_{a2} - y \\ \vdots \\ x_{an} - y \end{Bmatrix}, \qquad \{\ddot{y}\} = \begin{Bmatrix} \ddot{y} \\ \ddot{y} \\ \vdots \\ \ddot{y} \end{Bmatrix}$$

and dot representing differentiation with respect to time. Also,

$\alpha_a = m_a/m_p$, mass ratio.

$\zeta_a = c_a/2\sqrt{k_a m_a}$, fraction of critical damping in absorber.

$T_a = 2\pi\sqrt{m_a/k_a}$, undamped natural period of vibration of absorber.

$y = y_0 \sin \omega t$, sinusoidal base motion.

$\ddot{y} = -y_0\omega^2 \sin \omega t$, sinusoidal base acceleration.

$T_p = 2\pi\sqrt{m_p/k_p}$, undamped natural period of vibration of frame.

$\zeta_p = C_p/2\sqrt{k_p m_p}$, fraction of critical damping in frame.

A modified fourth order Runge–Kutta procedure has been used for the numerical solution of equation 2. Various combinations of the parameters of vibration absorbers as given in Table 1 were used along with the main frame data for the solution of equations. Acceleration response of various masses were calculated for various exciting frequencies. Response curves for acceleration ratio versus frequency ratio were plotted for main frame as well as for absorbers.

Results

Plot of amplification factor (μ_p) versus frequency ratio (η) for the frame alone is shown in fig. 5, where μ_p is defined as the ratio between maximum acceleration of the frame to that of the base, and η as ratio of exciting frequency to natural frequency of frame alone. It is seen that high amplification peak occurs at $\eta = 1$ which is termed as resonance.

Response curves for frame with one absorber are shown in figure 6. Curve 1 gives response of the frame showing two different peaks, one at about $\eta = 0.8$ and the other at $\eta = 1.2$. There is a sudden depression at $\eta = 1$ which was a very high peak in the earlier case when no absorber was used. This is because of the change in the natural frequency of vibration of the parent system by putting vibration absorber. Both peaks in this case have lesser amplification factor as compared to earlier case of no absorber.

In other cases two and three absorbers are respectively used along with the parent system. Response curves in these cases for frame and absorbers are shown in figures 7 and 8. In these cases the number of peaks increases, but because of the limitations in exciting frequency of the table, the full response curve could not be plotted. From these curves it is seen that amplitude of each peak is smaller than the amplitude in earlier cases. This shows that more number of absorbers are effective not only at one exciting frequency but at several other frequencies. Further, the peak amplitudes of parent system go down as the number of absorbers increases. Also from the response curves of absorbers, it is seen that peak amplitude of absorbers goes down as the number of absorbers increases.

Conclusions

The response of frame decreases with the use of vibration absorbers. The reduction is more if more number of absorbers are used Also the response of absorbers themselves decreases with the increase in number of absorbers. More number of absorbers are effective not only at one exciting frequency but at other frequencies also.

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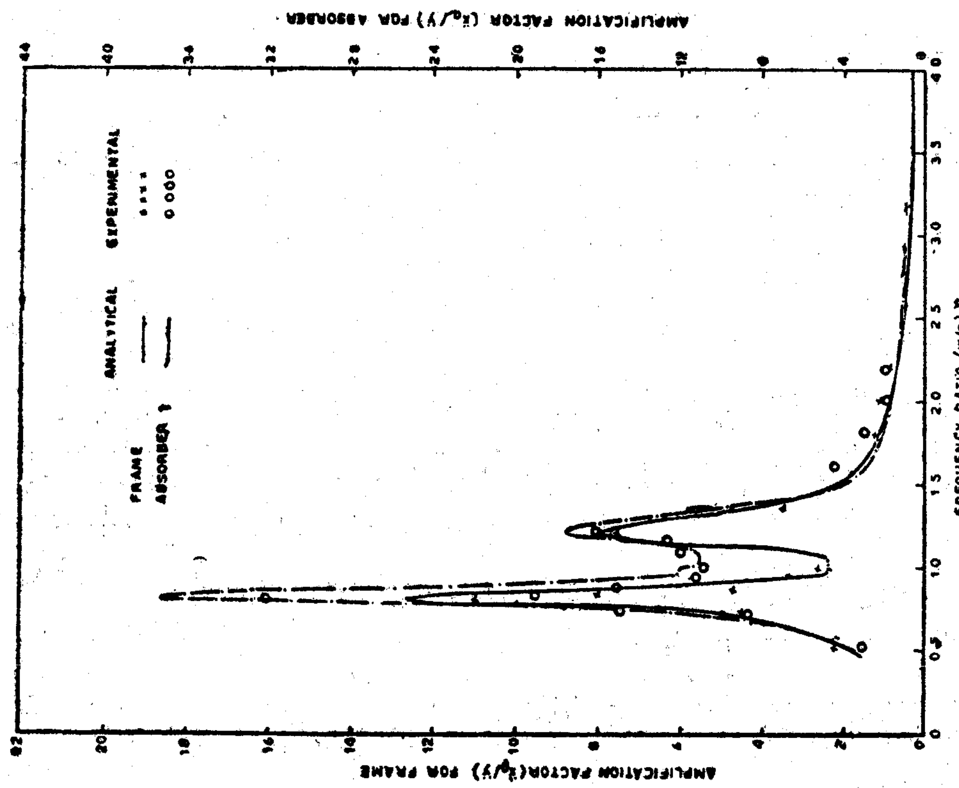

Fig. 6. Response Curvse for the Frame with Absorber 1

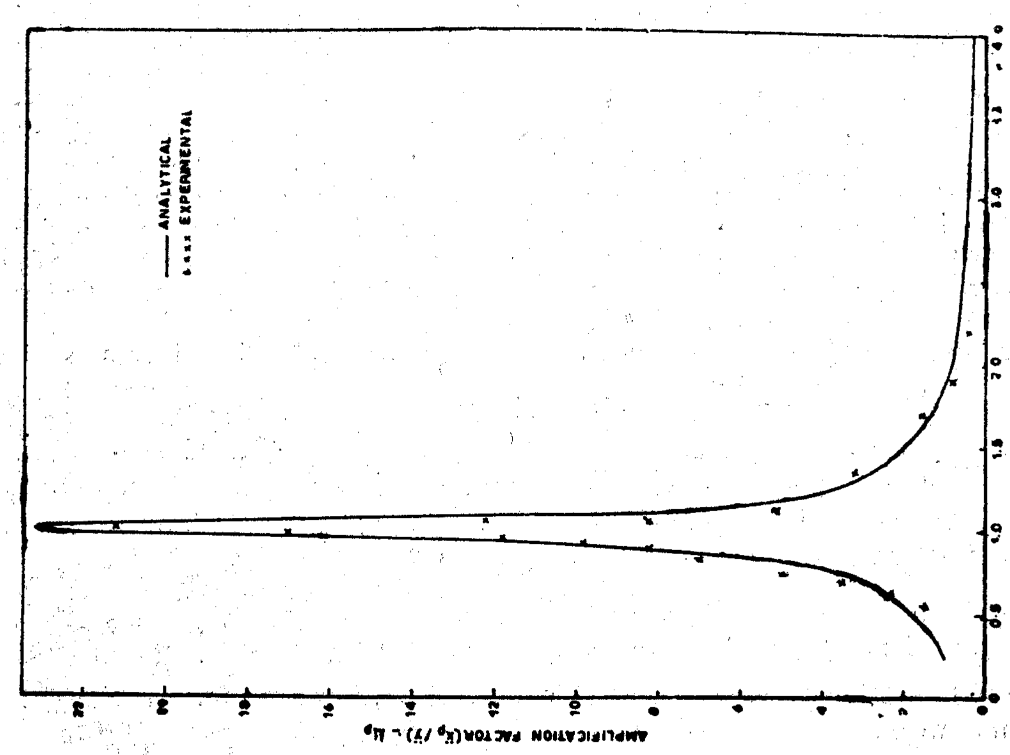

Fig. 5 Response Curve for Frame Alon: