COMPUTING PERIODS OF VIBRATION OF MULTISTOREYED BUILDING FRAMES

A. S. Arya*

SYNOPSIS

It is emphasised in this paper that in the dynamic analysis of multistoreyed buildings, the flexibility of girders must be considered along with the stiffness columns because assuming the girders rigid may lead to large errors in the calculated periods of vibration. A simplified method is suggested for computing the natural periods and the corresponding mode shapes taking the girder flexibility into account.

INTRODUCTION

Dynamic analysis of any structure subjected to impulsive or vibrating forces requires the knowledge of its stiffness and the damping characteristics. Stiffness determines the natural periods of vibration and the corresponding mode shapes almost exclusively since the effect of damping on natural periods is small and negligible. Damping has the influence of reducing the amplification factors under forced vibration. The effect is more pronounced when imposed period of vibration is close to the natural period, otherwise when forcing frequency is considerably less or more than the natural frequency, the effect of damping is small and negligible. The amplification of displacement and consequently of forces produced in the structure depends upon the ratios of forcing periods to the natural periods and the corresponding mode shapes. Therefore, determination of natural periods of vibration of a structure constitutes the most important step in the process of its dynamic analysis.

Multistoreyed buildings are very complex structures from the point of view of structural analysis especially under horizontal loads causing sidesway. This fact alone has resulted in the introduction of a large number of approximate methods of analysis for sidesway problems under static loads. Under dynamic loads the problem is all the more complicated because the natural periods of the building are found to be influenced to a great extent by the method of construction, floor slabs, wall panels, partitions, etc., which are otherwise assumed not to contribute to the stiffness of the building. How far they influence the stiffness and whether such contribution is of a permanent nature or may be destroyed in the very first shock given to the building is not known. It has been found (1)† that for existing multistoreyed buildings with steel frames, the measured fundamental period comes close to that computed by assuming building floors to be rigid, that is, having infinite moment of inertia as compared with the columns But for a bare frame or if the contribution to stiffness of filling material (which is not designed to carry forces due to shocks) is destroyed the stiffness is bound to decrease and actual girder stiffness must be considered for computing natural periods. Whereas assumption of rigid girders greatly simplifies the problem of analysis and much literature exists on this procedure, consideration of flexibility of girders complicates the problem because of consequent joint rotations. But this must be taken into account since the influence of girder flexibility on the natural periods is considerable. For example, for a particular 19-storey steel-frame building, the fundamental period was 1.3 sec. when girders were assumed rigid but 3.4 sec. when girder flexibility was considered(2). Figure (1) further shows the influence of girder flexibility on the fundamental periods of single bay multistoreyed frames (1). In the case of actual buildings, where beams are stiffened by the floor slabs, the margin between the two assumptions decreases considerably.

From the above discussion it follows that in dynamic computations stiffness properties of beams and

* Reader in Civil Engineering, University of Roorkee, Roorkee.

† Numbers in parentheses refer to the list of References at the end of the paper.

columns must be taken into account. Where a digital computer is available, such calculations may be done easily. But where such a facility is not present, this computation involves too much work. It is the aim of this paper to present a simplified method for calculating the fundamental period by Rayleigh's procedure, or all the periods and mode shapes if desired by the flexibility equations.

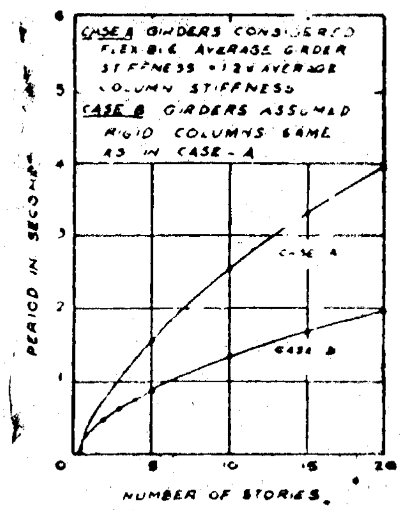


Fig. 1

METHOD OF ANALYSIS

It has been shown in (3), (4) and (5) that any general multistoreyed frame can be represented by a modified frame with single column. The stiffnesses in the single-column frame are modified in such a way that for the horizontal loads producing sideway of the original frame, the modified frame could be analysed as a non-sway frame. The sidesway deflections and moments in the modified frame are thus determined in a direct manner. From these, the moments in the original frame are then obtained by moment distribution done on the original frame. For determining the time-periods and corresponding mode shapes, the horizontal displacements are the only quantities required which can be obtained from the modified frame itself. Therefore no further reference to the original frame is necessary and hence the simplification. The modified frames for a few cases are shown in Fig. 2. In case (a) a two—column symmetrical frame is shown. The modified frame has its column twice as stiff as the individual columns of the original frame and the beams are twelve times as stiff as the corresponding beams. In this case the equivalence of the modified frame is "exact". The resulting deformations—slopes and displacements—of the modified frame will be exactly equal to those of the original frame without any approximation. In case (b) a frame obtained on the principle of multiples is shown together with the component one-bay frames which may be assumed to be making it. If the applied horizontal load is proportioned in the two component frames in the ratio 1:m, it is easily seen that the rotations and displacements of joints in the two component one-bay frames will be the same. Thus if the two adjacent columns of the one-bay frames are combined in one, neither compatibility nor equilibrium is disturbed. Therefore, for this frame also the column stiffness of modified frame is the sum of corresponding column stiffnesses of the original frame and the beam stiffness of the single column frame is 12 times the sum of all corresponding beam stiffnesses. The equivalence is again exact since the rotations of all the joints at any floor of such a frame are exactly equal. In case (c) a general frame is shown with irregular stiffness and nonuniform heights in different bays. It is well known that the joint rotations at any floor of this frame will in general be unequal. If the average value of all such rotations at any floor is considered, the general frame can be represented by a single column frame shown in Fig. 2 (c). In this case also the single column will have an aggregated stiffness of all columns in any storey but the factor with beam stiffnesses will be different from 12. A study of large number of frames (3) indicates that a factor 'A' may be used for the floors and 'A′' for the roof. Factors A and A′ are in general different from 12, but near to it. For certain frames their values may be taken from Table 1.

If the bases of the columns are hinged, the stiffness of single column of the modified frame becomes zero in the bottom storey as shown in Fig. 2 (d). The modified frame in cases (c) and (d) is not exactly equivalent to the original frame as its joint rotations are approximately equal to the average of joint rotations at any floor

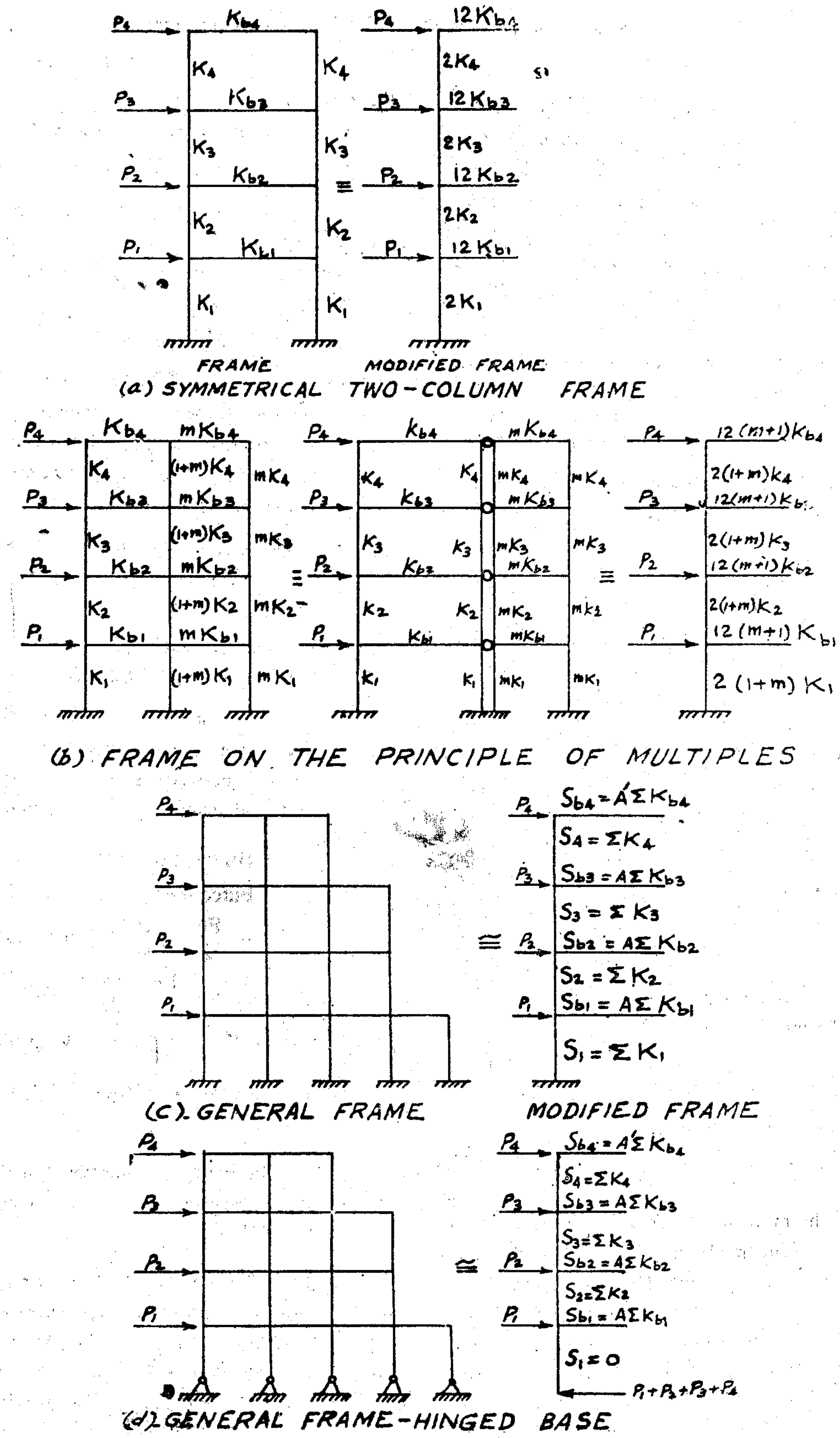


Fig. 2.—Modified Frames

of the original frame. Fortunately the effect of this approximation on the horizontal displacements of the frame is small. Therefore, the storey displacements and hence the periods and mode shapes may be computed with sufficient accuracy. If values of A and A′ are assumed according to Table 1, the results are expected to lie within ± 2% of the exact values.

TABLE 1.

Type of Frame	Factor A for Floors	Factor A′ for Roof
1. Symmetrical two column frames or frames on principle of Multiples	12.0	12.0
2. Frames approximately as in 1.	11.8	11.8
3. Irregular two column frame, column ratios on any floor not to exceed 4 to 1.	11.8	10.8
4. Irregular two column frame with higher ratios say 10 to 1 of column stiffnesses on any floor.	9.5	9.5
5. (a) Three column frame with same beam stiffnesses	11.9	11.4
(b) Many column frame with same beam stiffnesses	12.0	11.6
6. Four column frame with same beam stiffness in outer span and uniform in the interior spans of all storeys.	11.4	10.5
7. General Frame with irregular beam and column stiffnesses within ordinary limits.	11.5	11.5

For finding the moments in the single-column frame, the method of moment distribution is used with the modification that the carry over factor becomes '−1' instead of + ½. Then from the end moments of the column, the storey displacements, are determined immediately. The following example will illustrate the procedure.

Example 1.

To determine the horizontal displacements of frame shown in Fig. 3 (a)

Fig. 3 (b) shows the modified frame obtained by taking A and A′ equal to 11.5 as per case 7 of Table-1. The various steps in the calculation are as follows (Refer Fig. 3 a).

1. Distribution factors are determined as usual. Factors for column ends only need be written.
2. Initial moments are obtained by multiplying the storey shears by half of the storey heights that is, Fh/2. Here anticlockwise moments are taken negative.
3. Unbalanced joint moments are distributed and carried over as usual excepting that the carry-over factor is '−1'.
4. Column moments are found by summing the initial, the distributed and carry-over moments.
5. To obtain the relative storey displacements it was shown in Ref. (5) that the relative displacement Δ between the two ends of a storey may be found from the following relation:—

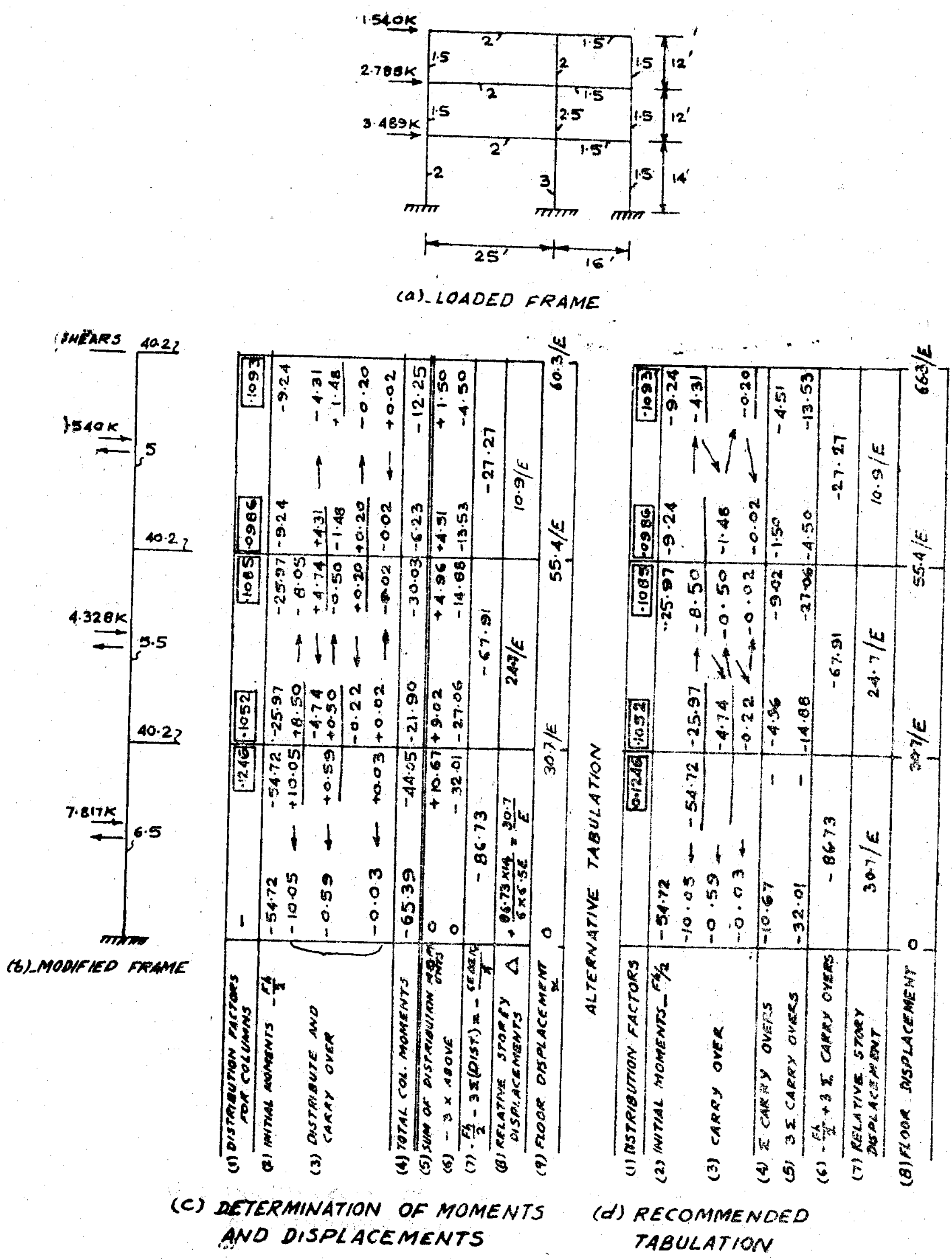
$$\frac{6ES\Delta}{h} = \text{(Initial end moment)} - (3\times \text{ sum of distributed moments at the ends of the storey)}$$
$$= -\frac{Fh}{2} - \sum (3\times \text{Dist. moments})$$
$$\Delta = \frac{h}{6ES}\left[-\frac{Fh}{2} - \sum(3\times \text{Dist. moments})\right] \quad (1)$$

where S is the column stiffness in the story.

This explains the calculations of Δ's through steps (5) to (8).

9. The floorwise displacements are obtained by summing up the relative storey displacements starting from the foundations.

Fig. 3 (d) shows an alternative tabulation in case only horizontal displacements are required as in t me-period calculation. Here the distributed mements are ommitted from writing. The carry over moments are written directly since these may be obtain d from the



1·540K
2·788K
3·489K
25'
16'
12'
12'
14'
SHEARS
(a) LOADED FRAME
(b) MODIFIED FRAME
ALTERNATIVE TABULATION
(c) DETERMINATION OF MOMENTS AND DISPLACEMENTS
(d) RECOMMENDED TABULATION

Fig. 3

unbalanced moments in proportion of the distribution factors and of the same sign as the unbalanced joint moments. The relative storey displacements may now be obtained from the following equation:—

$$\Delta = \frac{h}{6ES}\left[-\frac{Fh}{2} + (\Sigma\, 3 \times \text{Carryover moments})\right] \quad (2)$$

FUNDAMENTAL PERIOD

The fundamental time period of a structure may approximately be determined most easily by Rayleigh's method. In this method it is assumed that the mode shape is proportional to the deflection curve produced by a static load at each floor equal to mass times the acceleration due to gravity. When the building vibrates freely in the first mode, the horizontal displacement x_n at any floor 'n' at any time 't' is given by

$$x_n = CX_n \sin pt$$

where X_n is the mode shape at floor 'n'

C is a constant depending upon stiffnesses.

p is the fundamental natural frequency of free vibrations.

The mode shape is chosen to have the displacement at the top level X_h equal to 1. The maximum kinetic energy of mass m_n is given by

$$\tfrac{1}{2}\, m_n\, (CX_n p)^2$$

and the maximum potential energy is given by

$$\tfrac{1}{2} m_n\, g\, CX_n$$

Summing up the kinetic and potential energies separately and equating we get

$$\sum m_n (C_g X_n\, p)^2 = \sum m_n\, g\, C_g\, X_n$$

where C_g is the value of the constant obtained when horizontal load of 1g is considered in calculating static deflections. Putting $p = 2\,\pi/T$ (where T is the fundamental period) and solving

$$T = 2\pi \left[\frac{Cg}{g}\;\frac{\Sigma m_n X_n^2}{\Sigma m_n\; X_n}\right]^{\frac{1}{2}} \quad (3)$$

Example 2 given below illustrates the procedure of calculating fundamental period of a building by Rayleigh's method.

Example 2.

The four storeyed frame of Fig. 4 has the relative stiffnesses as shown. The actual stiffness of beams is K_o. The floor weights including weights of walls are $4w_o$, $4w_o$, $3w_o$ and $2w_o$ respectively. It is required to determine its fundamental period of vibration.

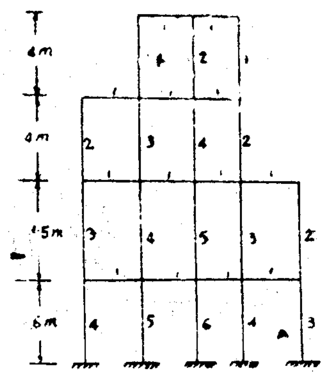


Fig. 4.

Using Rayleigh's method let us give a horizontal acceleration of g to the floor masses. Then the horizontal load applied to the frame at first floor level is

$$\frac{4w_o}{g} \times g = 4w_o$$

Similarly for the other floors the loads will be $4w_o$, $3w_o$ and $2w_o$. The deflections of floors are computed in Fig. 5. The modifying factors A and A′ are assumed 12 and 11.6 respectively.

Using the deflections camputed in Fig. 5, we get the approximate mode shape given in the last line adjusted in such a way that the displacement at the top is 1. This gives $C_g = 13.33 w_o/EK_o$

Using Equation 3 we get the time period

$$T = 2\pi \left[\frac{13.33 w_o}{gEK_o} \cdot \frac{4 \times .315^2 + 4 \times .628^2 + 3 \times .855^2 + 2 \times 1}{4 \times .315 + 4 \times .628 + 3 \times .855 + 2 \times 1}\right]^{\frac{1}{2}}$$

$$= 6.3 \sqrt{\frac{w_o}{EK_o}} \text{ seconds.}$$

TIME PERIODS AND MODE SHAPES

To determine all the time periods and mode shapes, the flexibility or the stiffness equations are used. In the present scheme, the flexibility equations are easily derived by obtaining influence lines of horizontal displacements. Let X_{nr} denote the displacement of r^{th} floor produced by a unit static load applied at .th floor level.

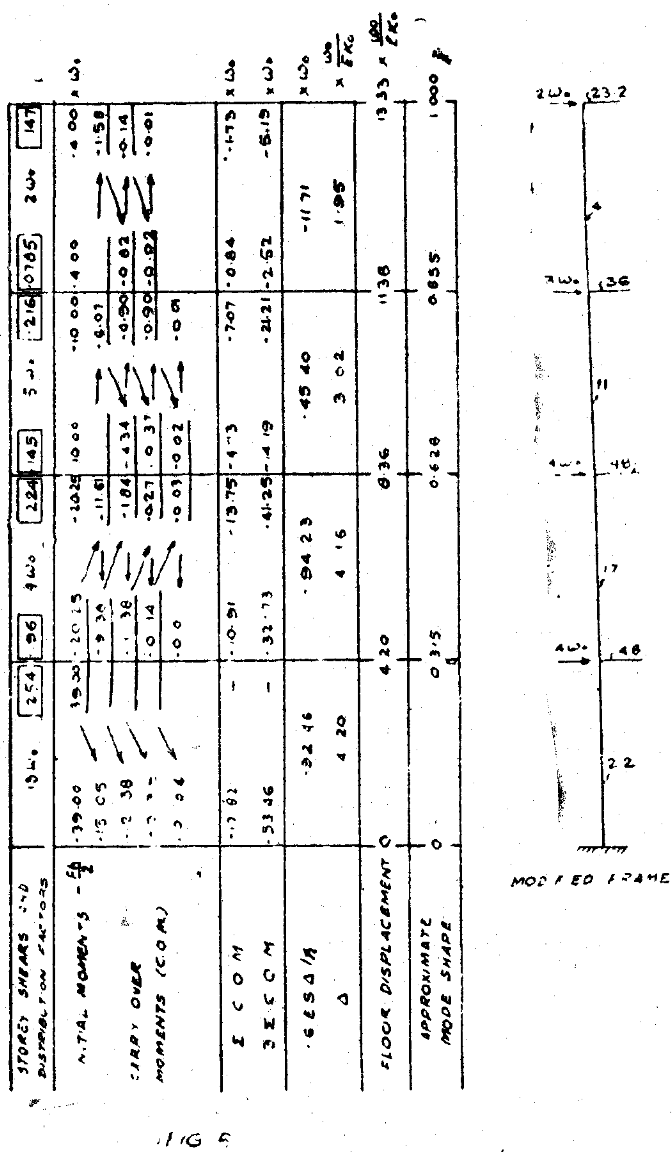


Fig. 5-

If the building is vibrating with a freuqency p, the inertia force applied at the r^{th} floor is $m_r\ p^2x_r$ where x_r denotes the dynamic displacement. Therefore, the flexibility equation for the n^{th} floor becomes,

$$x_n = p^2 \sum m_r\, x_r\, X_{nr}$$

By transposing

$$(1-p^2m_n\ X_{nr})\, x_n - p^2 \sum_{n \neq r} m_r x_r\, X_{nr} = 0 \qquad (4)$$

Such equations written for all the floors of a building give a set of homogeneous simultaneous algebraic equations in x_1, x_2, x_3 etc. For a possible non-zero solution, the determinant of the coefficients of x_1, x_2 etc. must vanish. This condition gives the characteristic equation for obtaining values of p and hance the mode shapes. The following example illustrates the prodedure.

Example 3.

All the periods and mode shapes for the frame of Fig. 4 will now be determined.

To obtain the floor displacements due to unit loads, we shall use the modified frame method. The calculations are shown in Fig. 6. Part (a) shows the modified frame assuming A = 12 and A′ = 11.6. The calculations for a unit load at first floor level are shown at (b). The initial moments for this case are

$$-1x\ 6/2 = -3.00 \text{ tm.}$$

The calculation of deflections follows the pattern of Example 1, Fig. 3 (d). The final numerical values of x's are to be multiplied by $1/EK_o$ and the results will be in tonne metre units.

The computations of deflections for unit loads applied at other floor levels are shown in Figs. 6 (c), (d) and (e). Since deflections obey the Reciprocal Law, it must be verified that the computed deflections are reciprocal. Results in (b), (c), (d) and (e) satisfy this condition. Now if p is the frequency of vibration of the building, we get the loads.

$$\frac{4w_o}{g}\ p^2x_1,\ \frac{4w_o}{g}\ p^2x_2,\ \frac{3w_o}{g}\ p^2x_3 \text{ and } \frac{2w_o}{g}\ p^2x_4$$

acting at the first, second, third floors and the roof respectively. Hence the compatibility equation at first floor may be written as follows:

$$x_1 = \frac{4w_o}{g}\ p^2x_1\ \frac{0.2453}{EK_o} + \frac{4w_o}{g}\ p^2x_2\ \frac{0.346}{EK_o} + \frac{3w_o}{g}\ p^2x_3\ \frac{0.366}{EK_o} + \frac{2w_o}{g}\ p^2x_4\ \frac{0.37}{EK_o}$$

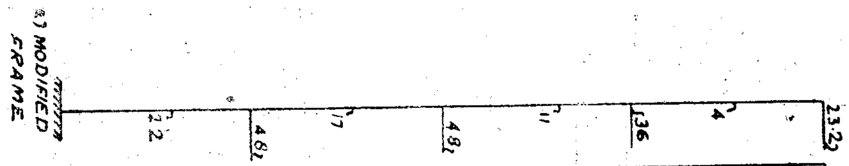
Let $\lambda = \frac{w_o p^2}{gEK}$. Therefore, we get for all the floors and roof the following equations ;

$$(1-0.981\lambda)\, x_1-1.384\lambda\, x_2-1.098\lambda\, x_3-0.74\lambda\, x_4=0 \quad \text{(i)}$$

$$-1.384\lambda\, x_1+(1-2.828\lambda)\, x_2-2.463\lambda x_3-1.688\lambda\, x_4=0 \quad \ldots \text{(ii)}$$

$$-1.464\lambda x_1-3.284\lambda x_2+(1-3.66\lambda)x_3-2.706\lambda x_4=0 \quad \text{(iii)}$$

$$-1.480\lambda x_1-3.376\lambda x_2-4.059\lambda x_3+(1-4.146\lambda)\, x_4=0 \quad \ldots \text{(iv)}$$



(b) UNIT LOAD AT FIRST FLOOR

DISTRIBUTION FACTORS		·254	·196	·224	·145	·216	·0765	·147	
INITIAL MOMENTS	-3.000	-3.000	–	–	–	–	–	–	
Σ C.O.M	-0.799	0	-0.142	-0.616	-0.019	-0.019	-0.001	-0.007	
Δ	0.2453		0.1003		0.020		0.004		$\times \frac{1}{EK_0}$
x	0	0.2453		0.3456		0.3656		0.3696	$\times \frac{1}{EK_0}$

(c) UNIT LOAD AT II FLOOR

INITIAL MOMENTS	-3.000	-3.000	-2.250	-2.250	–	–	–	–	
Σ C.O.M	-1.533	–	-0.783	-1.185	-0.113	-0.514	-0.005	-0.040	
Δ	0.345		0.361		0.114		0.0222		$\times \frac{1}{EK_0}$
x	0	0.346		0.707		0.821		0.843	$\times \frac{1}{EK_0}$

(d) UNIT LOAD AT III FLOOR

INITIAL MOMENTS	-3.000	-3.000	-2.250	-2.250	-2.000	-2.000	–	–	
Σ C.O.M	-1.683	–	-1.384	-1.300	-0.634	-0.898	-0.034	-0.231	
Δ	0.366		0.455		0.400		0.132		$\times \frac{1}{EK_0}$
x	0	0.366		0.821		1.221		1.353	$\times \frac{1}{EK_0}$

(e) UNIT LOAD AT ROOF

INITIAL MOMENTS	-3.000	-3.000	-2.250	-2.250	-2.000	-2.000	-2.000	-2.000	
Σ C.O.M	-1.715	–	-1.506	-1.325	-1.152	-0.977	-0.355	-0.419	$\times \frac{1}{EK_0}$
Δ	0.370		0.474		0.509		0.720		$\times \frac{1}{EK_0}$
x	0	0.370		0.844		1.353		2.073	$\times \frac{1}{EK_0}$

Fig. 6.

Multiplying equation (iii) by $m_3/m_1=3/4$ and equation (iv) by $m_4/m_1=1/2$, we shall see that the equations become symmetrical about the main diagonal. Thus rearrainging the equations we get

$$\left.\begin{array}{l} -(1-0.981\lambda)x+1.384\lambda x_2+1.098\lambda x_3+0.74\lambda x_4=0 \\ 1.384\lambda x_1-(1-2.828\lambda)\,x_2+2.463\lambda x_3+1.688\lambda x_4=0 \\ 1.098\lambda x_1+2.463\lambda x_2-(0.75-2.75\lambda)x_3+2.03\lambda x_4=0 \\ 0.74\lambda x_1+1.688\lambda x_2+2.03\lambda x_3-(0.5-2.073\lambda)\,x_4=0 \end{array}\right\} \quad (5)$$

The above equations are now in the standard form for determining the eigen values and the eigen modes. The equations are homogeneous, simultaneous algebraic equations and for a possible non-zero solution, the determinant of the coefficients of x's must be zero. This gives a fourth degree characteristic equation in λ the four roots of which are the eigen values. Knowing the eigen values the corresponding modes may the determined by substituting for λ in the obove equations and solving for x_2, x_3 and x_4 say by taking x_1 equal to 1. This procedure becomes involved as number of degrees of freedom increases. In that case, some numerical method must be used. Many such methods, such as Rayleigh quotient method, Holzer's procedure, ploting value of determinant against assumed λ values, method of intensification or successive approximations, etc. are available. In any case, the procedure for obtaining periods and modes after obtaining the compatibility equations is the same as used when the floors are assumed rigid and does not involve any new principles. As such it is left out of consideration here. Just for comparison, the following table gives the time periods and modes for the frame under consideration as computed from above wherein actual girder flexiblility has been considered and as calculated on the assumption of rigid floors.

TABLE

		Rayleighs' method	Actual Stiffnesses				Girders taken Rigid			
		Ist mode	Ist mode	2nd mode	3rd mode	4th mode	Ist mode	2nd mode	3rd mode	4th mode
Period $\times \frac{W_0}{EK_0}$ sec.		6.3	6.31	2.22	1.23	0.687	3.44	1.56	0.96	0.715
Mode	x_4	3.18	3.73	−3.06	0.90	−0.19	2.67	−2.44	0.66	−0.23
Shape	x_3	2.72	2.96	0.33	0.72	0.10	2.06	0.24	−1.25	+0.99
	x_2	2.00	2.12	1.82	−1.90	−0.39	1.59	1.07	0.07	−1.39
	x_1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

CONCLUSION

Time periods and mode shapes of multistoreyed buildings are appreciably different when flexibility of flour is neglected by assuming them rigid as compared with columns and when it is taken into account. Hence actual stiffness of members should be considered for dynamic calcutations. Such computations may be carried out with sufficient accuracy without spending too much labour and time by using the method suggested in the paper.

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SEISMOLOGICAL NOTES

A. N. Tandon*

Earthquakes felt in and near about India during July - September, 1963.

Date 1963	Origin Time (G.M.T.) h. m. s.	Epicentre Lat. (°N)	Epicentre Long. (°E)	Region	Approx. depth (Km)	Magnitude	Remarks
(1)	(2)	(3)		(4)	(5)	(6)	(7)
5 Jul.	07 19 15.8 (U.S.C.G.S.)	27.7	92.1	Assam	33	4.2	—
10 Jul.	02 11 56.3 (U.S.C.G.S.)	36.5	71.8	Hindukush	33	4.9	Recorded at a number of Indian observatories.
13 Jul.	19 08 40 (SHILLONG)	24.5	71	Sind, (West Pakistan)	—	5.6 (New Delhi)	Recorded at almost all Indian observatories.
	19 08 39.1 (U.S.C.G.S.)	24.9	70.3	Pakistan	33	—	—
14 Jul.	10 51 40 (SHILLONG)	36	72	Hindukush	—	5.9 (New Delhi)	Recorded at a number of Indian observatories.
	10 51 42.7 (U.S.C.G.S.)	36.1	70.6	Hindukush	120	5.1	—
14 Jul.	14 48 28.4 (U.S.C.G.S.)	30.3	78.5	Northern India	33	4.8	Recorded at a number of Indian observatories.
1 Aug.	15 32 15	Near Delhi		Delhi - Punjab border	—	—	Epi: about 42 Km to the NW of Delhi, felt at Sonepat (Punjab).
1 Aug.	22 51 15	Near Delhi		Delhi - Punjab border	—	—	Epi: about 41 Km. to the NW of Delhi, felt at Sonepat (Punjab).
3 Aug.	14 48 31	Near Delhi		Delhi - Punjab border	—	—	Epi: about 42 Km. to the NW of Delhi, felt at Sonepat (Punjab).
3 Aug.	19 42 18	Near Delhi		Delhi - Punjab border	—	—	Epi: about 41 Km. to the NW of Delhi, felt at Sonepat (Punjab).
11 Aug.	19 40 52.2 (U.S.C.G.S.)	36.2	71.2	Hindukush	148	—	Recorded at Delhi.

Date 1963	Origin Time (G.M.T.) h. m. s.	Epicentre Lat. (°N)	Epicentre Long. (°E)	Region	Approx. depth (Km)	Magnitude	Remarks
(1)	(2)	(3)		(4)	(5)	(6)	(7)
12 Aug.	18 29 38.8 (U.S.C.G.S.)	25.3	62.7	Near coast of west Pakistan	33	5.2	Recorded at a few Indian observatories.
13 Aug.	07 03 49.6 (U.S.C.G.S.)	36.6	70.9	Hindukush	244	4.7	Recorded at a few Indian observatories
22 Aug.	03 05 08	Near Delhi		Delhi - Punjab border	—	—	Epc. about 40 Km. to the NW of Delhi, felt at Sonipat (Punjab).
2 Sept.	01 34 30 (New Delhi)	34	75	Near Srinagar (Kashmir)	—	5.5	Major property of damage in Kashmir Valley.
	01 34 31.6 (U.S.C.G.S.)	33.9	74.7	Northern India	44	5.1	79 people killed and 400 hundred injured at Badgam near Srinagar.
2 Sept.	22 25 52 (U.S.C.G.S.)	26.2	90.0	Assam - India	220	—	Recorded at some Indian observatories.
6 Sept.	01 58 19	Near Delhi		Punjab - Delhi border	—	3.2	Epc. about 60 Km. North West of Delhi.
9 Sept.	21 41 43.6 (U.S.C.G.S.)	31.3	72.1	West Pakistan	33	4.7	Recorded at many Indian observatories.
12 Sept.	10 35 11 (P-time Delhi)	Near Delhi		Punjab - Delhi border	—	—	Epc. about 40 Km. NW of Delhi, felt at Sonipat (Punjab).
19 Sept.	16 31 15 (U.S.C.G.S.)	31.0	66.8	Afghanistan - Pakistan border	37	4.2	Recorded at some Indian observatories.
20 Sept.	15 01 44	—	—	North West Uttar Pradesh	—	—	Felt at Garbying.
24 Sept.	13 27 06	Near Delhi		Delhi - Punjab border	—	—	Epc. about 39 Km. to the NW of Delhi.
27 Sept.	21 57 40	Near Delhi		—	—	—	90 Km. away from Delhi.

* Director (seismology), Meteorological Observatory, New Delhi.