

## INTRODUCTION

Aseismic surface movements in seismically active regions of the earth during apparently quiet aseismic periods have attracted the attention of seismologists in recent years and the possibility of utilising them to obtain greater insight into the dynamics of earthquake processes, including accumulation of stress and strain, leading to seismic fault movements, has been recognised [Kasahara (1981)]. Some theoretical models of aseismic surface movements, in seismically active regions have been developed in recent years, and have been discussed by Mukherji and Mukhopadhyay (1984, 1986) and Cohen et al. (1984). In most of these theoretical models, a single locked or creeping fault is considered in models of the lithosphere-asthenosphere system. However, Mukherji and Mukhopadhyay (1984, 1986) have considered theoretical models with two interacting faults, and have explained the significance of fault, interacting in the process of stress accumulation and release in seismically active regions. In these theoretical models, the faults were taken to be both buried or both surface-breaking. Keeping in view the possibility of creep across faults of a fault system with buried as well as surface-breaking faults, a theoretical model of the lithosphere-asthenosphere system, with a surface-breaking fault and a buried fault, has been considered in this paper.

## FORMULATION

We consider a simple model of the lithosphere-asthenosphere system consisting of a visco-elastic half space with its material of the Maxwell type. We considered two long vertical and interacting strike-slip faults  $F_1$  and  $F_2$  in the half-space across which creep occurs under suitable conditions. We take one of the faults ( $F_1$ ) to be buried, and the other ( $F_2$ ) is taken to be surface-breaking. We introduce rectangular cartesian co-ordinates ( $y_1, y_2, y_3$ ) with the free surface as the plan  $y_3 = 0$  and the  $y_3$  — axis pointing into the half-space. We take the  $y_1$  —axis to be parallel to the plane of the faults. Then we can assume that the displacements, stresses and strains will be independent of  $y_1$ . We take the planes of the faults  $F_1$  and  $F_2$  be given by  $y_2 = 0$  and  $y_2 = D$  respectively. Let  $D_1$  be the depth of the lower edge of  $F_1$  below the free-surface. Also let  $d_2$  and  $D_2$  be the depths of the upper and lower edges of the fault  $F_2$  below the free surface. Fig 1 shows the section of the model by the plan  $y_1 = 0$ . For the model, since the displacements, stresses and strains are independent of  $y_1$ , we find that the displacement component  $u_1$  along the  $y_1$  axis and the stress components  $\tau_{12}$  and  $\tau_{13}$  associated with it are independent of the

other components of displacement and stress, and satisfy the relations,

$$\text{and } \left. \begin{aligned} \left( \frac{1}{\eta} + \frac{1}{\mu} \frac{\partial}{\partial t} \right) \tau_{13} &= \frac{\partial^2 u_1}{\partial t \partial y_3} \\ \left( \frac{1}{\eta} + \frac{1}{\mu} \frac{\partial}{\partial t} \right) \tau_{13} &= \frac{\partial^2 u_1}{\partial t \partial y_3} \end{aligned} \right\} \dots (1)$$

$$(-\infty < y_3 < \infty, y_3 \geq 0, t \geq 0)$$

where  $\mu$  is the effective rigidity and  $\eta$  is the effective viscosity, as in Mukherji and Mukhopadhyos (1984, 1986).

We consider the model during aseismic periods, leaving out the relatively small periods (if any) following sudden fault movements, when seismic disturbances are present in the model. For the slow, aseismic, quasi-static displacements we consider, the inertial forces are very small and are neglected. Hence the relevant stresses satisfy the relation

$$\frac{\partial}{\partial y_3} (\tau_{13}) + \frac{\partial}{\partial y_3} (\tau_{13}) = 0$$

$$(-\infty < y_3 < \infty, y_3 \geq 0, t \geq 0) \dots (2)$$

From (1) and (2), we find that

$$\frac{\partial}{\partial t} (\nabla^2 u_1) = 0 \dots (3)$$

which is satisfied if  $\nabla^2 u_1 = 0$

$$(-\infty < y_3 < \infty, y_3 \geq 0, t \geq 0).$$

At the free surface  $y_3 = 0$ , we have the boundary condition

$$\tau_{13} = 0, \text{ on } y_3 = 0$$

$$(t \geq 0, -\infty < y_3 < \infty) \dots (4)$$

We assume that the tectonic forces maintain a constant shear stress far, away from the faults, while stresses near the fault may change with time due to fault movement (including fault creep). We then have the boundary conditions,

$$\tau_{13} \rightarrow 0 \text{ as } y_3 \rightarrow \infty$$

$$(\text{for } -\infty < y_3 < \infty, t \geq 0). \dots (5)$$

$$\text{and } \tau_{12} \rightarrow \tau_0 \text{ as } y_2 \rightarrow \infty$$

$$\text{for } y_3 \geq 0, t \geq 0) \dots (6)$$

## DISPLACEMENTS AND STRESSES IN THE ABSENCE OF FAULT MOVEMENT

In the absence of any movement across the faults, the displacement and stresses are continuous throughout the model. In this case, we measure the time  $t$  from the instant at which the relations (1)—(6) become valid

for the model. Let  $(u_1)_0, (\tau_{12})_0, (\tau_{13})_0$ , which may be functions of  $(y_2, y_3)$ , be the values of  $(u_1), (\tau_{12}), (\tau_{13})$  at  $t = 0$ .  $(u_1)_0, (\tau_{12})_0, (\tau_{13})_0$  also satisfy the relations (1)–(6).

The initial and boundary value problem is solved, following exactly the same method as in Mukherji and Mukhopadhyay (1984, 1986), using Laplace transforms, to obtain

$$\left. \begin{aligned} u_1(y_2, y_3, t) &= (u_1)_0 + \frac{\tau_{\infty} y_3 t}{\eta} \\ \tau_{12}(y_2, y_3, t) &= (\tau_{12})_0 e^{-\frac{\mu t}{\eta}} + \tau_{\infty} \left(1 - e^{-\frac{\mu t}{\eta}}\right) \\ \tau_{13}(y_2, y_3, t) &= (\tau_{13})_0 e^{-\frac{\mu t}{\eta}} \end{aligned} \right\} \dots (7)$$

As in Mukherji and Mukhopadhyay (1984, 1986), we find from (7), that if the shear stress  $\tau_{12}$  near the faults, tending to cause strike-slip movements is less than  $\tau_{\infty}$  at  $t = 0$ , then there would be a continuous accumulation of shear stress near the faults for  $t > 0$ , with  $\tau_{12} \rightarrow \tau_{\infty}$  near the faults, as  $t \rightarrow \infty$ .

We assume, as in Mukherji and Mnkhopadhyas (1984, 1986) that aseismic creep commences across  $F_1$  and  $F_2$  when  $\tau_{12}$  reaches critical values  $\tau_c$  and  $\tau_c'$  respectively near  $F_1$  and  $F_2$ .

If  $\tau_c$  or  $\tau_c' < \tau_{\infty}$ , then seismic creep would commence across  $F_1$  or  $F_2$  after a finite time. From (7), we find that the fault across which aseismic creep commences first would be determined by the values of  $\tau_c, \tau_c'$  and the value of  $(\tau_{12})_0$  near the faults  $F_1$  and  $F_2$ . Assuming that  $\tau_c, \tau_c' < \tau_{\infty}$ , we consider next the situation after aseismic creep commences across  $F_1$  or  $F_2$  or both.

### DISPLACEMENTS AND STRESSES AFTER THE COMMENCEMENT OF FAULT CREEP

If fault creep commences across  $F_1$  or  $F_2$  or both, the relations (1) – (6) are still satisfied together with the following conditions of creep across  $F_1$  and  $F_2$

$$\left. \begin{aligned} [u_1]_1 &= U_1(t_1) f_1(y_3) H(t_1) && \dots (8a) \\ \text{across } F_1 &(0 \leq y_3 \leq D_1, y_2 = 0) \end{aligned} \right\}$$

$$\text{and } \left. \begin{aligned} [u_1]_2 &= U_2(t_2) f_2(y_3) H(t_2) && \dots (8b) \\ \text{across } F_2 &(d_2 \leq y_3 \leq D_2, y_2 = D) \end{aligned} \right\}$$

where  $t_1 = t - T_1$ ,  $t_2 = t - T_2$ .

$$[u_1]_1 = Lt \quad [u_1] \quad - \quad Lt \quad [u_1]$$

$$y_2 \rightarrow 0+0 \quad y_2 \rightarrow 0-0$$

is the relative displacement across  $F_1$  corresponding to the fault creep and  $U_1(t_1) = 0$  for  $t \leq 0$  i.e.,  $t \leq T_1$ , so that

$$[u_1] = 0 \text{ for } t \leq T_1.$$

The velocity of creep across  $F_1$  is

$$\frac{\partial}{\partial t} [u_1]_1 = V_1(t_1) f_1(y_2)$$

where  $V_1(t_1) = \frac{d}{dt_1} [U_1(t_1)]$

Again  $[u_1]_2 = Lt \quad (u_1) \quad - \quad Lt \quad (u_1)$   
 $y_2 \rightarrow D+0 \quad y_2 \rightarrow D-0$   
 $(d_2 \leq y_2 \leq D_2)$

is the relative displacement across  $F_2$  corresponding to the fault creep and  $U_2(t_2) = 0$  for  $t_2 \leq 0$  i.e.,  $t \leq T_2$ . so that  $[u_1]_2 = 0$  for  $t \leq T_2$

The velocity of creep across  $F_2$  is

$$\frac{\partial}{\partial t} [u_1]_2 f_2(y_2) = V_2(t_2)$$

where  $V_2(t_2) = \frac{d}{dt_2} [U_2(t_2)]$ .

$T_1$  and  $T_2 (\geq 0)$  are the times of commencement of creep across  $F_1$  and  $F_2$  respectively. In case no creep occurs at any time across  $F_1$  or  $F_2$ , we simply take

$$U_1(t_1) = 0 \text{ for all } t \geq 0$$

or  $U_2(t_2) = 0$  for all  $t \geq 0$ ,

So that  $[u_1]_1 = 0$  or  $[u_1]_2 = 0$  for all  $t \geq 0$ .

considering the model after the commencement of fault creep, across  $F_1$  or  $F_2$ , or both, we try to find solutions for  $u_1, \tau_{12}, \tau_{13}$  in the form :

$$\left. \begin{aligned} u_1 &= (u_1)_1 + (u_1)_2 + (u_1)_3 \\ \tau_{12} &= (\tau_{12})_1 + (\tau_{12})_2 + (\tau_{12})_3 \\ \text{and } \tau_{13} &= (\tau_{13})_1 + (\tau_{13})_2 + (\tau_{13})_3 \end{aligned} \right\} \dots\dots (9)$$

where  $(u_1)_1, (\tau_{12})_1, (\tau_{13})_1$  are continuous everywhere in the model, satisfy the relations (1) — (6, and have the values  $(u_1)_0, (\tau_{12})_0, (\tau_{13})_0$  at  $t = 0$ , while  $(u_1)_2, (\tau_{12})_2, (\tau_{13})_2$  are zero for  $t \leq T_1$ , satisfy (1) — (5), (8a) and are continuous everywhere except across  $F_1$  satisfying the following condition which replaces (6) :

$$(\tau_{12})_2 \rightarrow 0 \text{ as } [y_2] \rightarrow \infty \quad (y_2 \geq 0, t \geq t_1) \dots\dots (10a)$$

Again,  $(\tau_{12})_2, (u_1)_2, (\tau_{13})_2$  are zero for  $t \leq T_2$ , satisfy (1) — (5), (8b) and are continuous everywhere except across  $F_2$ , satisfying the following condition which replaces (6) :

$$(\tau_{12})_2 \rightarrow 0 \text{ as } [y_2] \rightarrow \infty \text{ (} y_2 \geq 0, t \geq T_2 \text{)} \quad \dots\dots (11b)$$

In this case, it is clear that the solutions (9) will satisfy (1) - (6), (8a) and (8b). We note that  $(u_1)_1, (\tau_{12})_1, (\tau_{13})_1$  satisfy exactly the same conditions as those satisfied by  $u_1, \tau_{12}, \tau_{13}$  in the absence of fault creep movement. Hence,  $(u_1)_1, (\tau_{12})_1, (\tau_{13})_1$  have the same expressions as those for  $u_1, \tau_{12}, \tau_{13}$  given in (7).

On substituting  $t_1 = t - T_1$ , we find that  $(u_1)_2, (\tau_{12})_2, (\tau_{13})_2$  which are functions of  $(t_1, y_2, y_3)$  satisfy the following relations, obtained from (1)-(5) (10), (8a) and (8b)

$$\left( \frac{1}{\eta} + \frac{1}{\mu} \frac{\partial}{\partial t_1} \right) (\tau_{12})_2 = - \frac{\partial^2 (u_1)_2}{\partial t_1 \partial y_2}$$

$$\left( \frac{1}{\eta} + \frac{1}{\mu} \frac{\partial}{\partial t_1} \right) (\tau_{13})_2 = \frac{\partial^2 (u_1)_2}{\partial t_1 \partial y_3} \quad \dots\dots (1a)$$

$$\frac{\partial}{\partial y_2} (\tau_{12})_2 + \frac{\partial}{\partial y_3} (\tau_{13})_2 = 0 \quad \dots\dots\dots (2a)$$

$$\nabla^2 (u_1)_2 = 0 \quad \dots\dots\dots (3a)$$

[(1a), (2a), (3a) being valid for  $-\infty < y_2 < \infty, y_3 \leq 0, t_1 \geq 0$ ]

$$(\tau_{12})_2 = 0 \text{ as } y_2 = 0 \quad (-\infty < y_2 < \infty, t_1 \geq 0) \quad \dots\dots\dots (4a)$$

$$(\tau_{12})_2 \rightarrow 0 \text{ as } y_2 \rightarrow \infty \quad (-\infty < y_2 < \infty, t_1 \geq 0) \quad \dots\dots\dots (5a)$$

$$(\tau_{13})_2 \rightarrow 0 \text{ as } y_3 \rightarrow \infty \quad (y_3 \geq 0, t_1 \geq 0) \quad \dots\dots\dots (6a)$$

$$[(u_1)_2] = u_1(t_1) f_1(y_2) \text{-across } F_1$$

$$(y_2 = 0, 0 \leq y_3 \leq D_1).$$

and  $\dots\dots\dots (7a)$

$$(u_1)_2, (\tau_{12})_2, (\tau_{13})_2 = 0 \text{ for } t_1 \leq 0$$

To obtain solutions for  $(u_1)_2, (\tau_{12})_2, (\tau_{13})_2$  for  $t_1 \leq 0$ , we take Laplace transforms of (1a) - (7a) with respect to  $t_1$ .

This gives a boundary value problem which can be solved by using a suitably modified form of a Green's function technique developed by Maruyama (1966), as explained by Mukherji and Mukhopadhyay (1984, 1986). On inverting the Laplace transforms, as in Mukherji and Mukhopadhyay (1984, 1986), we

$$(u_1)_s = H(t-T_1) \frac{U_1(t_1)}{2\pi} \Psi_{11}(y_2, y_3) \dots\dots(11)$$

$$(\tau_{12})_s = H(t-T_1) \left( \frac{\mu}{2\pi} \right) \int_0^t v_1(\tau) e^{-\frac{\mu(t_1-\tau)}{\eta}} \phi_{11}(y_2, y_3) d\tau \dots\dots(12)$$

$$(\tau_{13})_s = H(t-T_1) \left( \frac{\mu}{2\pi} \right) \int_0^t v_1(\tau) e^{-\frac{\mu(t_1-\tau)}{\eta}} \phi_{s1}(y_2, y_3) d\tau \dots\dots(13)$$

( $y_2 \neq 0$ )

where  $t_1 = t - T_1$ ,  $H(t - T_1)$  is the Heaviside function, so that  $H(t - T_1) = 0$  for  $t \leq T_1$  and  $H(t - T_1) = 1$  for  $t > T_1$

$$\Psi_{11}(y_2, y_3) = \int_0^{D_1} f_1(x_3) \left[ \frac{y_3}{(x_3 - y_3)^2 + y_3^2} + \frac{y_3}{(x_3 + y_3)^2 + y_3^2} \right] dx_3 \dots\dots(14)$$

$$\phi_{11}(y_2, y_3) = \frac{\partial \Psi_{11}}{\partial y_2} = \int_0^{D_1} f_1(x_3) \left[ \frac{(x_3 - y_3)^2 - y_3^2}{\{(x_3 - y_3)^2 + y_3^2\}^2} + \frac{(x_3 + y_3)^2 - y_3^2}{\{(x_3 + y_3)^2 + y_3^2\}^2} \right] dx_3 \dots\dots(15)$$

and  $\phi_{s1}(y_2, y_3) = \frac{\partial \Psi_{11}}{\partial y_3} = \int_0^{D_1} 2f_1(x_3) \left[ \frac{(x_3 - y_3)y_3}{\{(x_3 - y_3)^2 + y_3^2\}^2} - \frac{(x_3 + y_3)y_3}{\{(x_3 + y_3)^2 + y_3^2\}^2} \right] dx_3 \dots\dots(16)$

( $y_2 \neq 0$ )

In (11) - (16),  $U_1(t_1)$  and  $V_1(t_1)$  vanish for  $t_1 < 0$  i.e.,  $t < T_1$ . We now consider the functions  $(u_1)_s$ ,  $(\tau_{12})_s$ ,  $(\tau_{13})_s$ . These functions satisfy the relations (1a) - (6a), with  $t_1$  replaced by  $t_2$ , together with the following relation.

$$[u_1]_s = U_2(t_2) f_2(y_3) \text{ across } F_2$$

$$(d_2 \leq y_3 \leq D_2, y_2 = D) \dots\dots(7b)$$

replacing (7a).  $(u_1)_s, (\tau_{12})_s, (\tau_{13})_s = 0$  for  $t_s \leq 0$ .

Now following the same procedure as in the determination of  $(u_1)_s, (\tau_{12})_s, (\tau_{13})_s$ , we obtain the expressions for  $(u_1)_s, (\tau_{12})_s, (\tau_{13})_s$  as follows :-

$$\begin{aligned}
 (u_1)_s &= H(t - T_s) \frac{U_s(t_s)}{2\pi} \Psi'_{11}(y'_s, y_s) \\
 (\tau_{12})_s &= H(t - T_s) \left(\frac{\mu}{2\pi}\right) \left[ \int_0^{t_s} V_s(\tau) e^{-\frac{\mu}{\eta}(t_s - \tau)} d\tau \right] \phi'_{11}(y'_s, y_s) \\
 (\tau_{13})_s &= H(t - T_s) \left(\frac{\mu}{2\pi}\right) \left[ \int_0^{t_s} V_s(\tau) e^{-\frac{\mu}{\eta}(t_s - \tau)} d\tau \right] \phi'_{21}(y'_s, y_s) \\
 (y'_s \neq 0, \text{ i.e., } y_s \neq D) & \dots\dots (17)
 \end{aligned}$$

where  $\Psi'_{11}(y'_s, y_s) = \int_{d_s}^{D_s} f_s(x_s) \left[ \frac{y'_s}{(x_s - y'_s)^2 + y_s'^2} + \frac{y'_s}{(x_s + y'_s)^2 + y_s'^2} \right] dx_s$  ..... (18)

$$\begin{aligned}
 \phi'_{11}(y'_s, y_s) &= \int_{d_s}^{D_s} f_s(x_s) \left[ \frac{(x_s - y'_s)^2 - y_s'^2}{\{(x_s - y'_s)^2 + y_s'^2\}^2} + \frac{(x_s + y'_s)^2 - y_s'^2}{\{(x_s + y'_s)^2 + y_s'^2\}^2} \right] dx_s \dots\dots (19)
 \end{aligned}$$

$$\begin{aligned}
 \phi'_{21}(y'_s, y_s) &= \int_{d_s}^{D_s} f_s(x_s) \left[ \frac{2y'_s(x_s - y'_s)}{\{(x_s - y'_s)^2 + y_s'^2\}^2} - \frac{2y'_s(x_s + y'_s)}{\{(x_s + y'_s)^2 + y_s'^2\}^2} \right] dx_s \dots\dots (20) \\
 (y'_s \neq 0) &
 \end{aligned}$$

where  $y'_s = y_s - D, y_s' = y_s$ .

Hence finally we obtain complete expressions for  $u_1, \tau_{12}, \tau_{13}$  from (9), where  $(u_1)_1, (\tau_{12})_1, (\tau_{13})_1$  have the same expressions as those for  $u_1, \tau_{12}, \tau_{13}$  given in (7).  $(u_1)_s, (\tau_{12})_s, (\tau_{13})_s$  are given by (11) - (16), and  $(u_1)_s, (\tau_{12})_s, (\tau_{13})_s$  are given by (17) - (20).

The shear strain  $e_{12}$  is given by

$$e_{12} = \frac{\partial u_1}{\partial y_2} = (e_{12})_0 + \frac{\tau \infty t}{\eta} + H(t - T_1) \frac{U_1(t_1)}{2\pi} \phi'_{11}(y_2, y_3) + H(t - T_2) \frac{U_2(t_2)}{2\pi} \phi'_{11}(y_2, y_3) \dots (21)$$

( $t_1 = t - T_1$ ,  $t_2 = t - T_2$ ,  $y_2 = y_2 - D$ ,  $y_3 = y_3$ ),

The integrals for  $\Psi_{11}$ ,  $\phi_{11}$ ,  $\phi_{21}$ ,  $\Psi'_{11}$ ,  $\phi'_{11}$ ,  $\phi'_{21}$  in (14) — (20) can be obtained in closed form if  $f_1(y_3)$ ,  $f_2(y_3)$  are polynomials. In particular, if the relative displacement due to creep is independent of depth, so that

$$f_1(y_3) = \text{constant} = K \text{ (say)}$$

$$(0 \leq y_3 \leq D_1),$$

$$\text{and } f_2(y_3) = \text{constant} = K' \text{ (say)} \quad (d_2 \leq y_3 \leq D_2).$$

we find, as in Mukherji and Mnkhopadyay (1984, 1986), that  $\phi_{11}$ ,  $\phi_{21}$ ,  $\phi'_{11}$ ,  $\phi'_{21}$  (and hence  $\tau_{12}$ ,  $\tau_{13}$ ,  $\tau'_{12}$ ,  $\tau'_{13}$ ) have singularities at the lower edge of the surfacebreaking fault and at the upper and lower edges of the buried faults. Following exactly the method mentioned in Mukherji and Mukhopadyay (1984, 1986) we find that the integrals  $\Psi_{11}$ ,  $\phi_{11}$ ,  $\phi_{21}$ ,  $\Psi'_{11}$ ,  $\phi'_{11}$ ,  $\phi'_{21}$  are finite everywhere in the model, including the edge of faults, for all finite ( $y_2, y_3, t$ ), provided the following conditions are all satisfied simultaneously by  $f_1(y_3)$  and  $f_2(y_3)$  :

(A) Conditions to be satisfied by  $f_1(y_3)$  :

(i)  $f_1(y_3)$  and  $f_1'(y_3)$  are continuous in  $0 \leq y_3 \leq D_1$

(ii)  $f_1''(y_3)$  is either continuous in  $0 < y_3 < D_1$ , or has a finite number of points of finite discontinuity in  $0 < y_3 < D_1$ .

(iii) either  $f_1''(y_3)$  is finite and continuous at  $y_3 = 0$  and  $y_3 = D_1$  or there exist real finite constant  $m$  and  $n$ , both  $< 1$ ,  $(y_3)^m f_1''(y_3) \rightarrow 0$  or to a finite limit as  $y_3 \rightarrow 0 + 0$  ..... (22A) and  $(D_1 - y_3)^n f_1''(y_3) \rightarrow 0$  or to a finite limit as  $y_3 \rightarrow D_1 - 0$ .

(iv)  $f_1(D_1) = 0$ ,  $f_1'(D_1) = 0 = f_1'(0)$ .

These conditions imply that the magnitude of the relative displacement across the fault varies smoothly with depth and approaches zero with sufficient smoothness as  $y_3 \rightarrow D_1 \rightarrow 0$  at the lower edge.

(B) Conditions to be satisfied by  $f_2(y_3)$  :

(i)  $f_2(y_3)$ ,  $f_2'(y_3)$ , are continuous in  $d_2 \leq y_3 \leq D_2$



(i)  $f_2(d_2) = f_2(D_2) = f_2'(d_2) = f_2'(D_2) = 0.$

(iii)  $f_2''(y_3)$  is either continuous in  $d_2 < y_3 < D_2$

or has a finite number of points of finite discontinuity in  $d_2 < y_3 < D_2$   
 and (iv) either  $f_2''(y_3)$  is finite and continuous at (22B)

$y_3 = d_2$  and  $y_3 = D_2,$

or there exist real finite constants  $m$  and  $n$  both  $< 1,$   
 such that

$(y_3 - d_2)^m f_2(y_3) \rightarrow 0$

or to a finite limit as  $y_3 \rightarrow d_2 + 0$

and  $(D_2 - y_3)^n f_2''(y_3) \rightarrow 0$

or to a finite limit as  $y_3 \rightarrow D_2 \rightarrow 0.$

If  $f_1(y_3)$  and  $f_2(y_3)$  satisfy the above conditions, in order to facilitate evaluation of the integrals, we carry out integration by parts of the integrals in the expressions for displacement, stress and strain and express the integrals in the following forms:-

$$\Psi_{11}(y_2, y_3) = - \int_{-y_3}^{D_1-y_3} f_1'(y_3 + y) \tan^{-1} \left( \frac{y}{y_2} \right) dy - \int_{y_3}^{D_1+y_3} f_1(z - y_3) \tan^{-1} \left( \frac{z}{y_2} \right) dz \quad \dots (23)$$

$$\phi_{11}(y_2, y_3) = - \frac{1}{2} \left[ \int_{-y_3}^{D_1-y_3} f_1''(y+y_3) \log_e (y^2 + y_2^2) dy + \int_{y_3}^{D_1+y_3} f_1''(z - y_3) \log_e (z^2 + y_2^2) dz \right] \quad \dots (24)$$

$$\phi_{21}(y_2, y_3) = - \int_{-y_3}^{D_1 - y_3} f_1'(y_3 + y) \tan^{-1} \left( \frac{y}{y_2} \right) dy - \int_{y_3}^{D_1+y_3} f_1'(z - y_3) \tan^{-1} \left( \frac{z}{y_2} \right) dz \quad \dots (25)$$

$$\Psi'_{11}(y_2', y_3') = - \int_{d_2 - y_3'}^{D_2 - y_3'} f_2'(y_3' + y) \tan^{-1} \left( \frac{y}{y_2'} \right) dy$$

$$- \int_{d_2 + y_3'}^{D_2 + y_3'} f_2(z - y_3) \tan^{-1} \left( \frac{z}{y_2'} \right) dz \quad \dots (26)$$

$$\phi'_{11}(y_2', y_3') = - \frac{1}{2} \left[ \int_{d_2 - y_3'}^{D_2 - y_3'} f_2''(y + y_3) \log_e (y^2 + y_2'^2) dy \right.$$

$$\left. + \int_{d_2 + y_3'}^{D_2 + y_3'} f_2''(z - y_3) \log_e (z^2 + y_2'^2) dz \right] \quad \dots (27)$$

$$\phi'_{21}(y_2', y_3') = - \int_{d_2 - y_3'}^{D_2 - y_3'} f_2''(y_3' - y) \left[ \tan^{-1} \left( \frac{y}{y_2'} \right) \right] dy$$

$$- \int_{d_2 + y_3'}^{D_2 + y_3'} f_2''(z - y_3') \left[ \tan^{-1} \left( \frac{z}{y_2'} \right) \right] dz \quad \dots (28)$$

The primes in  $f_1'(y_3 + y)$ ,  $f_1''(z - y_3)$ ,  $f_2'(y_3' + y)$ ,  $f_2''(z - y_3)$  etc. denote differentiation with respect to the argument. The integrals, in (23)-(28) can be expressed in closed form in terms of elementary functions if  $f_1(y_3)$ ,  $f_2(y_3)$  are polynomials satisfying (22a) and (22b). Otherwise, they can be evaluated approximately without difficulty by numerical integration if  $f_1(y_3)$ ,  $f_2(y_3)$  satisfy (22A) and (22B), but are not polynomials.

#### DISCUSSION OF THE RESULTS AND CONCLUSIONS :

To study in greater detail the changes of displacements, stresses and strains and strains near the faults with time, and specially the influence of faults creep, we compute the changes of the surface displacement  $u_1$  and surface shear strain  $e_{12}$  near the faults, as well as the shear stress  $\tau_{12}$  near the faults, tending to cause strike-slip movement, for relevant values of the model parameters  $\mu$ ,  $\eta$ ,  $D_1$ ,  $D_2$ ,  $D$ ,  $\tau_{\infty}$  and for relevant types of creep velocities the faults, Keeping in view the case of shallow strike-slip faults in the lithosphere, we take values for  $\mu$  in the range  $(3 \text{ to } 4) \times 10^{11}$  dynes/cm<sup>2</sup>,

while  $D_1, D_2, D, d_2$  are taken to have values in the range 5 to 40 kms, with  $d_2 > D_2$ . The computations have been carried out for some simple types of fault creep, where the creep displacements across  $F_1$  and  $F_2$  are given by

$$\left[ u_1 \right] = V_1 t_1 f_1 (y_3) H (t - T_1)$$

across  $F_1$

$$\text{and } \left[ u_1 \right] = V_2 t_2 f_2 (y_3') H (t - T_2)$$

across  $F_2$ ,

where  $V_1, V_2$  are constants, so that creep velocities do not change with time, and  $f_1 (y_3), f_2 (y_3')$  are polynomials, satisfying the conditions (22A) and (22B) for finite displacements, stresses and strains, so that  $u_1, e_{12}, \tau_{12}, J_{12}$  are finite for all finite  $(y_3, y_3', t)$  in the model. In particular, we consider the case in which

$$\left. \begin{aligned} f_1 (y_3) &= 1 - \frac{3y_3^2}{D_1^2} + \frac{2y_3^3}{D_1^3} \\ \text{and} \\ f_2 (y_3') &= \frac{16(y_3' - d_2)^2 (D_2 - y_3')^2}{(D_2 - d_2)^2} \end{aligned} \right\} \dots\dots(29)$$

For  $V_1$  and  $V_2$ , we consider values in the range 0 to 5 cms/year, which is the range of observed creep velocities on the surface across creeping strike-slip faults in North America. For  $\eta$ , we consider values in the range  $10^{21}$ — $10^{22}$  poise, keeping in view the fact that Cathles (1976) has obtained results on post-glacial uplift and corresponding theoretical results for theoretical models of the lithosphere-asthenosphere system with values of  $\mu$  in this range. For  $\tau_{\infty}$ , we consider values in the range 50 to 200 bars, and for  $(\tau_{12})_0$  near the faults we consider values in the range 0 to 100 bars ; with  $(\tau_{12})_0 < \tau_{\infty}$  in all cases.

The computed values of displacements stresses and strains show that, in the absence of fault creep, there is a gradual accumulation of the shear strain  $e_{12}$  and the shear stress  $\tau_{12}$  near the faults, increasing the possibility of a strike-slip fault movement. The rate of accumulation of  $\tau_{12}$  decreases slowly with time, and ultimately  $\tau_{12} \tau_{\infty}$  if no fault creep occurs. But if creep occurs  $F_1$  or  $F_2$  or both, it has a significant influence on the displacements, stresses and strains near  $F_1$  and  $F_2$ , where the nature of the effect of fault creep depends on the following factors :-

- (a) the creep velocities and their variation with depth [i.e.,  $V_1, V_2, f_1(y_3), f_2(y_3)$ ]
- (b) the depths and dimensions of the faults and the distance between them [ i.e.,  $D_1, d_2, D_2, D$  ]
- (c) the relative positions of the faults,
- (d) the values of  $\mu$  and  $\eta$ , the model parameters related to material rheology,
- (e) the displacements, stresses and strains present at  $t=0$ ,
- (f) the shear stress  $\tau_{00}$  far away from the faults, maintained by tectonic forces.

From the computed values, it is found that, when the creep across a fault commences, it reduces the rate of accumulation of surface shear strain and shear stress near itself. For sufficiently large creep velocities across a fault, there is a continuous aseismic release of shear stress near the fault under suitable circumstances, so that the possibility of a sudden fault movement, generating an earthquake is reduced continuously.

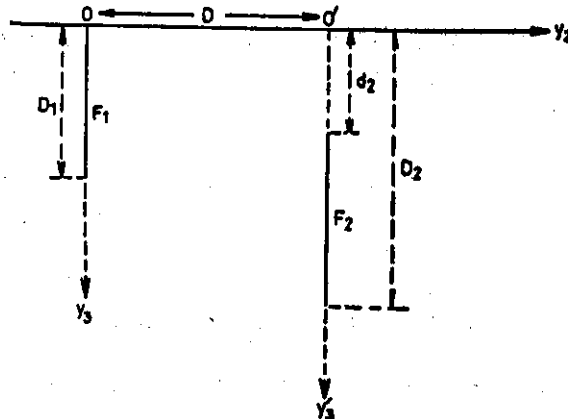


Fig. 1. Section of the model by the plane  $y_1=0$

The effect of creep across one fault on the shear stress and strain near the other depends significantly on the relative positions of the faults and also on their depths and dimensions and the distance between them. Considering the effect of creep across  $F_1$  or  $F_2$ , we find that if  $F_2$  lies in a particular region of the model, which we call  $f_1$ , creep across  $F_1$  increase the rate of accumulation of shear stress near  $F_2$ . But if  $F_2$  lies in two others regions of the model which we call  $R_1$  and  $R_2$ , the creep across  $F_1$  reduces the rate of accumulation of shear stress near  $F_2$ . These regions are shown in Fig. 2.



$V_1(t_1), V_2(t_2)$  ] are constants the magnitude of the effect of fault creep across  $F_1$  and  $F_2$  on the shear stresses, are found to be proportional to the creep velocities. If the creep velocities change with time, the relation is more complex. But, in this case also, an increase in the creep velocity across a fault leads to an increase in the magnitude of the effect of creep on the shear stresses near the fault itself and near the other fault.

On computing the surface shear strain, whose changes with time can be monitored by repeated geodetic surveys and instrumental observations, we find that, in our model, fault creep has a significant effect on the surface shear strain ( $\epsilon_{12}$ )  $y_3=0$ . For the surface breaking fault  $F_1$ , it is found that fault creep across  $F_1$  reduces the rate of accumulation of surface shear strain in the regions above  $F_1$  and  $F_2$ . But for the buried fault  $F_2$ , it is found that creep across  $F_2$  results in increase in the rate of accumulation of the surface shear strain ( $\epsilon_{12}$ )  $y_3=0$  in a region  $E_1'E_2'$  vertically above  $F_2$ , and in its reduction outside this region, which is shown in Fig.3. The width of  $E_1'E_2'$  depends on the fault parameters  $d_2, D_2$  and  $f_2$  ( $y_3'$ ) for  $F_2$ , and  $E_1'E_2'$  is symmetrical about the line on the surface  $y_3=0$  vertically above  $F_2$ .

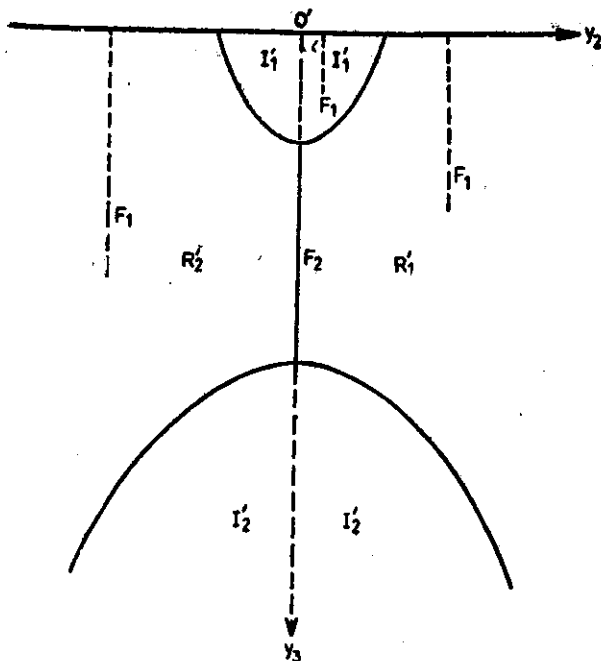
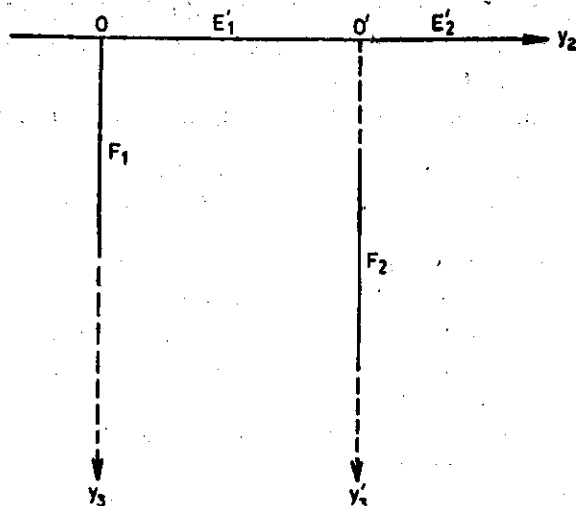


Fig. 3 : Creep across  $F_2$ -region of increase in the rate of shear stress accumulation ( $I_1, I_2$ ) and regions of decrease in the rate of shear stress accumulation ( $R_1, R_2$ )



**Fig. 4 :** Region E<sub>1</sub>, E<sub>2</sub> of increase in the rate of accumulation of surface shear strain due to creep across F<sub>1</sub>

Fig. 4 shows the changes in the shear strain  $e_{12}$  on the surface above the faults and the shear stress  $\tau_{12}$  near the faults, tending to cause strike-slip fault movements, for some typical values of the model parameters, which are given as follows :-

$$\eta = 10^{12} \text{ poise,}$$

$$\mu = 3 \times 10^{11} \text{ dynes/cm}^2,$$

$$(\tau_{12})_0 = 25 \text{ bars in the region of the faults,}$$

$$[u_1] = v_1 t_1 f_1(y_3') H(t - T_1) \text{ across } F_1$$

$$\text{and } [u_1] = v_2 t_2 f_2(y_3') H(t - T_2) \text{ across } F_2,$$

where  $f_1(y_3)$ ,  $f_2(y_3')$  are given by (29), and  $v_1$ ,  $v_2$  are in the range 0 to 5 cms/year. In the first situation,  $F_1$  and  $F_2$  are in different parallel planes, and have similar dimensions. The configuration is shown in Fig. 5. In the second situation,  $F_1$  is vertically above  $F_2$ . This configuration is shown in Fig. 6. Fig. 7 shows the changes with time of the quantity.

$$E_{12} = [e_{12} - (e_{12})_0] y_3 = 0 \times 10^7.$$

(=change in surface shear strain  $\times 10^7$ ), at points vertically above  $F_1$  and  $F_2$ , in the different planes as in Fig. 5,

$$\text{with } D_1 = D_2 = D = 15 \text{ Kms,}$$

$$\text{and } d_2 = 5 \text{ kms,}$$

$$T_1 = 50 \text{ years,}$$

$$T_2 = 100 \text{ years,}$$

$$v_1 = v_2 = 2 \text{ Cms/year.}$$

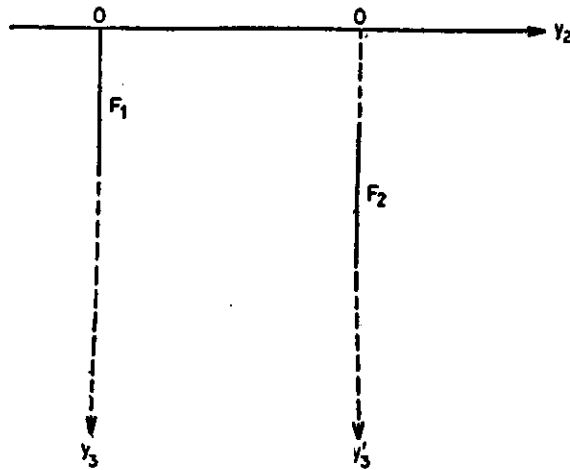


Fig. 5 :  $F_1$  and  $F_2$  in different parallel planes with approximately similar dimensions and at levels which do not differ appreciably.

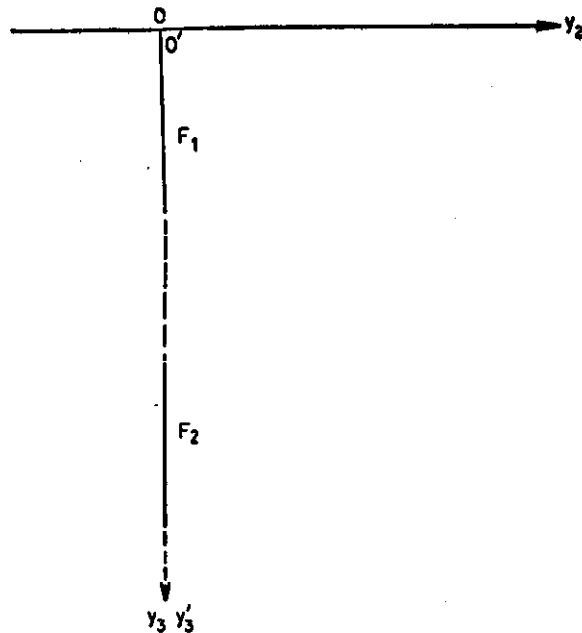
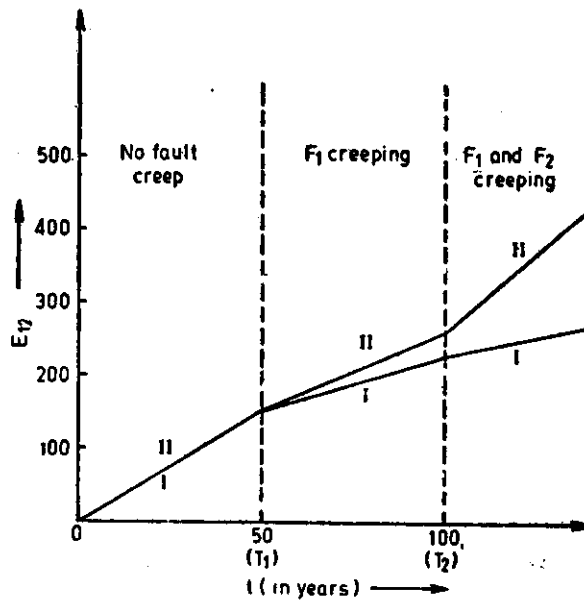


Fig. 6 :  $F_1$  vertically above  $F_2$





**Fig. 7 :** Changes in the surface shear strain with time above  $F_1$  and  $F_2$  which are in different parallel planes as in Fig. 5. The lines I and II correspond to points vertically above  $F_1$  and  $F_2$  respectively.

The lines I and II show the changes in  $E_{12}$  with the time  $t$  at surface points vertically above  $F_1$  and  $F_2$  respectively. Creep across  $F_1$ , which starts earlier in this case, is found to decrease the rate of accumulation of surface shear strain near  $F_1$  itself, and to a lesser extent, above  $F_2$ . Subsequent creep across  $F_2$ , in this case, is found to increase the rate of accumulation of surface shear strain vertically above  $F_2$  itself, while decreasing further the rate of accumulation of surface shear strain above  $F_1$ .

Fig 8 shows the variations in the same quantity  $E_{12}$  with time, in the case in which  $F_2$  is vertically above  $E_2$ , as in Fig. 6 with  $D_1=15$  kms,  $d_2=20$  kms,  $D_2=35$  kms,  $D=0$ ,  $V_1=V_2=2$  cms/year,  $T_1=50$  year, and  $T_2=100$  years. on the surface  $y_3=0$  vertically above  $F_1$  and  $F_2$ . It is found, in this case, that creep across  $F_1$  decreases the rate of accumulation of surface shear strain above  $F_1$  and  $F_2$ . But subsequent creep across  $F_2$  again increases the rate of accumulation of surface shear strain above  $F_1$  and  $F_2$ , although this compensates only partially for the effect of creep, across  $F_1$ , which tends to decrease this rate.

Fig. 9 shows the variations of the quantity

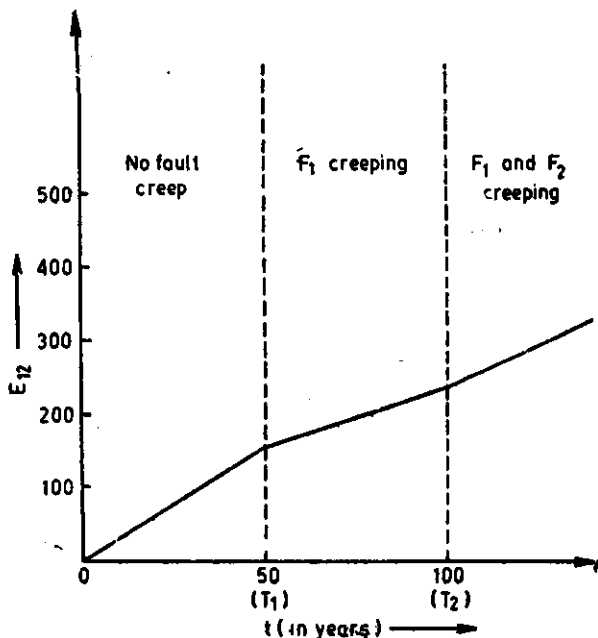


Fig. 8 : Changes in the surface shear strain above  $F_1$  and  $F_2$  with time in the case in which  $F_1$  is vertically above  $F_2$  as in Fig. V. 6

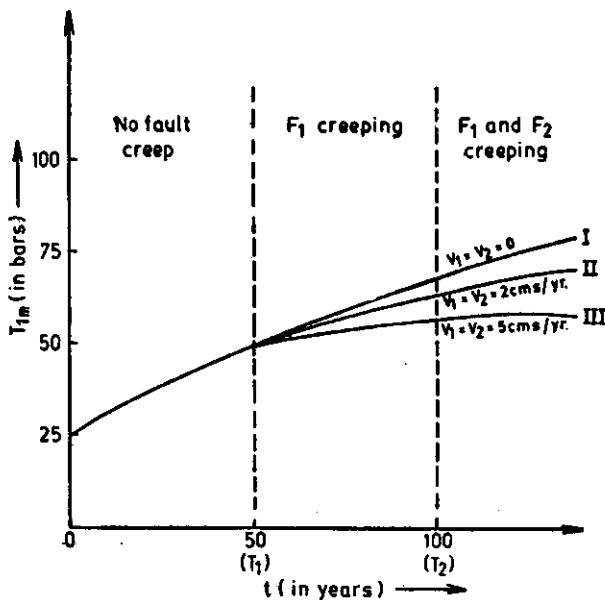


Fig. 9 : Changes in  $T_{1m}$ , the maximum shear stress near  $F_1$ , with time, where  $F_1$  and  $F_2$  are in different parallel planes, as in Fig. 5.

$$T_{1m} = \max [(\tau_{12})_{y_2 \rightarrow 0}, 0 \leq y_3 \leq D_1]$$

with time, in the case in which  $F_1$  and  $F_2$  are in different parallel planes, with  $D_1 = D_2 = D = 15$  kms,  $d_3 = 5$  kms,  $T_1 = 50$  years and  $T_2 = 100$  years.

The curves I, II and III in Fig. 9 corresponds to  $V_1 = V_2 = 0$  (no fault creep),

$V_1 = V_2$  cms/year and

$V_1 = V_2 = 5$  cms/year respectively.

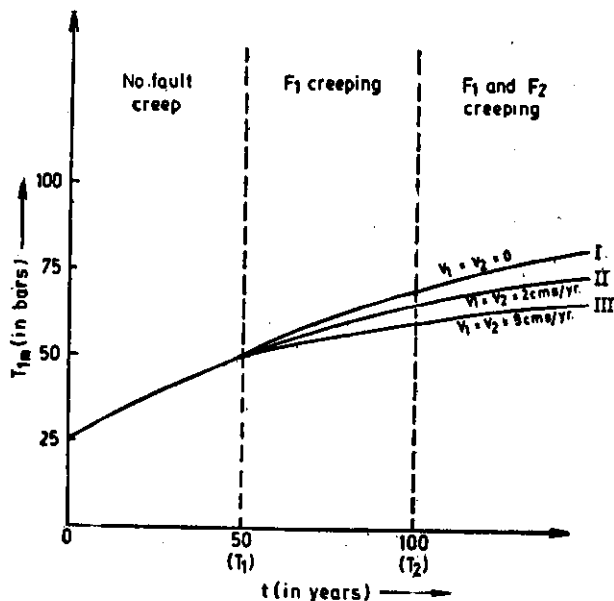


Fig. 10 . Changes in  $T_{1m}$ , the maximum shear stress near  $F_2$ , with time, where  $F_1$  and  $F_2$  are in different parallel planes, as in Fig. 5.

Fig. 10 also corresponds to exactly the same model, with the same values of  $D_1$ ,  $D_2$ ,  $D$ ,  $d_3$ ,  $T_1$  and  $T_2$ , and it shows the variation of time of  $T_{2m} = \max [(\tau_{12})_{y_2 \rightarrow 0}, d_3 \leq y_3 \leq D_2]$ .

In Fig. 10, the curves I, II and III corresponds to  $V_1 = V_2 = 0$ ,  $V_1 = V_2 = 2$  cms/year and  $V_1 = V_2 = 5$  cms/year respectively. Here  $T_{1m}$ , and  $T_{2m}$  which are functions of  $t$ , represent at any time  $t$  the maximum values of the shear stress  $\tau_{12}$  near the faults  $F_1$  and  $F_2$  respectively at that time. Fig. 9 and Fig. 10, we find that, in the absence of fault creep, there is a steady accumulation of shear stress near  $F_1$  and  $F_2$ . The rate of increase of shear stress falls off slowly with time. This accumulation of shear stress increases the

possibility of a sudden seismic fault movement. Creep across each fault reduces the rate of accumulation of shear stress near itself, and to a lesser extent near the other fault, in the case considered here. For the case  $V_1=V_2=5$  cms/year, when both the faults are creeping for  $t > T_2$ , the cumulative effect of creep across the two faults is found to lead to a steady aseismic release of the maximum shear stress near  $F_1$  and  $F_2$ , so that the possibility of a sudden seismic fault movement is progressively reduced. The model to which Fig 9 and Fig. 10 corresponds is the same as the model for which Fig. 7 shows the variations of the surface shear strain above  $F_1$  and  $F_2$ , with the same values of the model parameters.

In Fig. 11 and Fig. 12, we have shown the variations with time of the quantities  $T_{1m}$  and  $T_{2m}$  respectively, defined earlier, in the case in which  $F_1$  is vertically above  $F_2$ , as in Fig. 6 with  $D_1=15$  kms,  $d_2=20$  kms,  $D=35$  kms,  $D=0$ ,  $T_1=50$  years and  $T_2=100$  years. In this case, creep across  $F_1$  or  $F_2$  tends to release the shear stress  $\tau_{12}$  near the fault itself, but tends to increase the rate of shear stress accumulation near the other neighbouring fault.

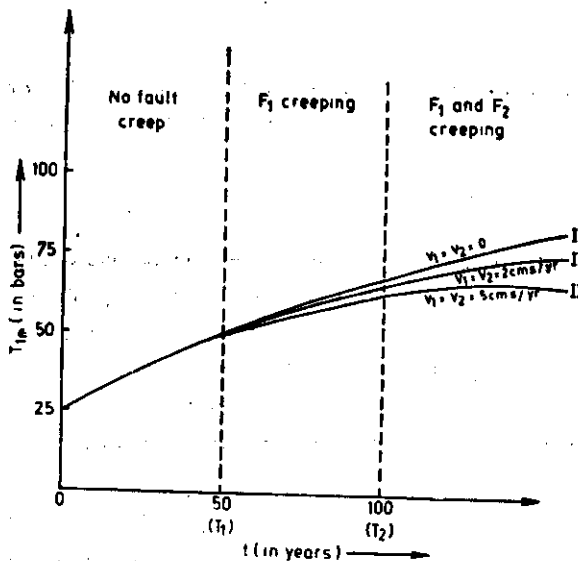


Fig. 11 ; Changes with time of  $T_{1m}$ , the maximum shear stress near  $F_1$ , with  $F_1$  vertically above  $F_2$ , as in Fig. 6.

In conclusion, we may say, that we find that the effect of fault creep on a fault system consisting of a buried fault and a surface-breaking fault depends significantly on the relative positions of the faults, and may lead

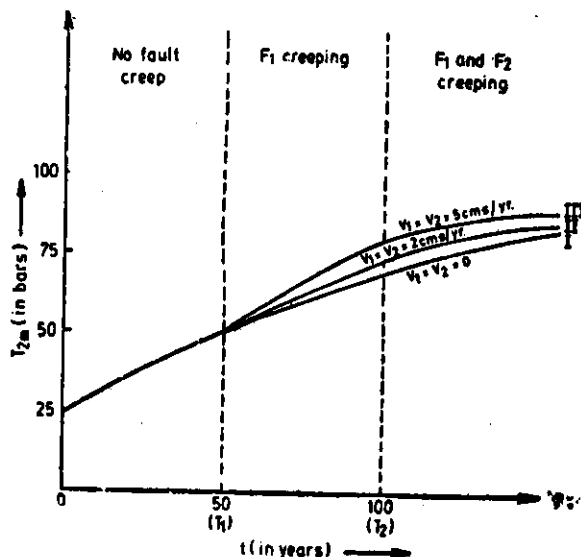


Fig. 12 : Changes with time of  $T_{2m}$ , the maximum shear stress near  $F_1$  is vertically above  $F_2$ , as in Fig. V. 6.

to aseismic stress accumulation or stress release depending on their mutual positions, as well as other model parameters, including creep velocity.

#### ACKNOWLEDGEMENT

The authors thank Prof. D. K. Sinha, Pro-Vice Chancellor, University of Calcutta for his kind interest in the research work. One of the authors (Purabi Mukherji) acknowledges the financial assistance received under the DSA Programme of the U. G. C. in the Department of Mathematics, Jadavpur University. The authors also thank the Computer Centre of the University of Calcutta for the computational facilities made available for the work presented in this paper.

#### REFERENCES

- (1) Cohen, S. C. and Kramer, M. J. (1984) : "Crustal Deformation, the Earthquake Cycle and models of Visco-elastic Flow in the Asthenosphere", Geophys. J. Roy. Astr. Soc., Vol. 78, pp. 735-750.
- (2) Kasahara, K. (1981) : "Earthquake Mechanics", Cambridge Univ. Press.

- (3) Maruyama, T. (1966): "On two dimensional dislocations in an infinite and semi-infinite medium", Bull Earthquake Res. Inst. Tokyo Univ., 44 (Part 3), pp 811-871.
- (4) Mukherji, P. and Mukhopadhyay, A. (1984): "On two interacting creeping vertical surface-breaking strike-slip faults in the lithosphere", Bull. ISET, Vol. 21, pp. 163-191.
- (5) Mukherji, P. and Mukhopadhyay, A. (1986): "Two aseismically creeping and interacting buried vertical strike-slip faults in the lithosphere", Bull. ISET, Vol. 23, pp. 91-117.