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INELASTIC RESPONSE OF ONE-STOREY ASYMMETRIC SYSTEMS TO BI-DIRECTIONAL SPATIAL EARTHQUAKE GROUND MOTIONS

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ABSTRACT

The inelastic response of a one-storey system with two-way eccentricities and subjected to bi-directional spatial earthquake ground motion is analyzed in this paper. 20 sets of bi-directional spatially varying horizontal earthquake ground motion time histories are numerically simulated for the analysis. The simulated motions are compatible individually with Newmark-Hall design response spectrum with 5% damping and normalized to 0.5g, and are compatible with an empirical coherency loss function between each other. Ensemble mean responses of the system to 20 sets of ground motions are estimated. Effects of system parameters such as uncoupled torsional-to-lateral frequency ratios, stiffness eccentricities in both directions, as well as the spatial ground motion wave passage effect, on coupled inelastic torsional-lateral responses are investigated. Numerical results are presented in dimensionless form. They are also compared with the code torsional provisions. For comparison purpose, some results obtained with linear elastic analysis are also presented.

KEYWORDS: Torsional Response, Eccentricity, Spatially Varying Ground Motion, Inelastic Response

INTRODUCTION

Coupled inelastic torsional-lateral responses of single storey asymmetric systems to earthquake ground motions have been a subject of investigation of many researchers (Kan and Chopra, 1977; Hejal and Chopra, 1989; Rutenberg and Pekau, 1987; Chandler and Duan, 1991; Duan and Chandler, 1993). In those studies, only uni-directional ground motion and structural eccentricity in one direction were considered. The effects of system parameters such as coupled torsional-to-lateral frequency ratios and stiffness eccentricity on structural responses and design code coefficients were evaluated. Studies on the coupled responses of one-way eccentric (Riddell and Santa-Maria, 1999) or two-way eccentric (Hao, 1998) system to bi-directional earthquake ground motions were reported. It was concluded that the effect of bi-directional ground motion on coupled torsional-lateral responses is significant. Recently, much effort has been spent on methods for controlling the torsional responses of structures (Jangid and Datta, 1997; Singh. et al., 2002).

Besides structural eccentricity, torsional responses of a structure can also be induced by torsional ground motion or by spatially varying ground motions at various structural supports. In early studies of torsional responses of structures to torsional base motion, torsional base motions were estimated by considering the wave passage effect of the travelling seismic wave, while the loss of coherency between motions at various points owing to wave propagation was not considered (Newmark, 1969; Morgan et al., 1983). In more recent studies of coupled torsional-lateral responses to torsional base motions, recorded time histories at various points in a building were used to estimate torsional base motions (De La Llera and Chopra, 1994a, 1994b, 1995).

Torsional responses of one-storey systems to spatially varying ground motions have also been investigated. In those studies, the structural system considered is a symmetric (Veletsos and Prasad, 1989) or asymmetric (Hahn and Liu, 1994) rigid deck rested on a circular rigid foundation; or a symmetric (Hao and Duan, 1996), one-way eccentric (Hao and Duan, 1995) or a two-way eccentric (Hao, 1998) square rigid plate supported by four columns. Either random vibration method or time history analyses were used. In all those studies, only linear elastic response analyses were performed. Except one study that analyzed the two-way eccentric system to bi-directional ground motions (Hao, 1998), all the above studies considered only uni-directional spatially varying ground motions. It should be noted that earthquake ground motion spatial variation effect is less significant, if a structure is supported by more columns owing to the averaging effects by columns on ground motion spatial variations. The model of circular deck resting on a rigid foundation is equivalent to a deck supported by infinite number of columns; thus, it results in the least significant spatial variation effects on structural responses. On the other hand, a rigid deck supported by four columns will induce the most pronounced ground motion spatial variation effects on structural responses.

All the previous studies revealed that coupled torsional-lateral responses depend strongly on the uncoupled torsional-to-lateral vibration frequency ratio of the system, and on structural eccentricity and ground motion spatial variations. Torsional response usually induces torque but reduces storey shear. It might also increase storey shear, if the system has significant two-way eccentricities (Hao, 1998). Based on linear elastic analysis of a two-way eccentric one-storey system to bi-directional spatially varying ground motions (Hao, 1998), it was found that, although the spatial variation of seismic motion over a building base is not significant, its effect on coupled torsional-lateral responses could be more significant than structural eccentricity effect on torsional response, if the system is torsionally flexible and has small eccentricity. However, structural eccentricity effect is usually more pronounced if the system is torsionally stiff, and/or has large eccentricity.

As a structure is usually designed to respond inelastically during strong shaking for economic and energy dissipation purposes, it is necessary to analyze the inelastic responses of two-way eccentric systems to bi-directional spatially varying earthquake ground motions in order to have a more realistic and thorough understanding of the problem. This paper is an extension of the work done by the first author previously, based on linear elastic assumption (Hao, 1998). Here, inelastic responses of the same one-storey, two-way eccentric, square rigid plate, supported by four columns and subjected to bi-directional spatially varying ground motions, are calculated by time history analysis. Similarly, 20 sets of bi-directional, spatially varying ground motion time histories are simulated as input for 20 independent analyses. The simulated motions are individually compatible with Newmark-Hall design spectrum with 5% damping and normalized to 0.5g, and are compatible with an empirical coherency loss function between each other. Ensemble means and standard deviations of coupled torsional-lateral responses are calculated. For comparison purposes, a few linear elastic results are also calculated and presented. Effects on coupled torsional-lateral responses of system parameters, such as uncoupled torsional-to-lateral vibration frequency ratio, stiffness eccentricity in both directions, as well as the ground motion phase shift, are investigated. Numerical results are presented through dimensionless parameters. They are also compared with code torsional provisions.

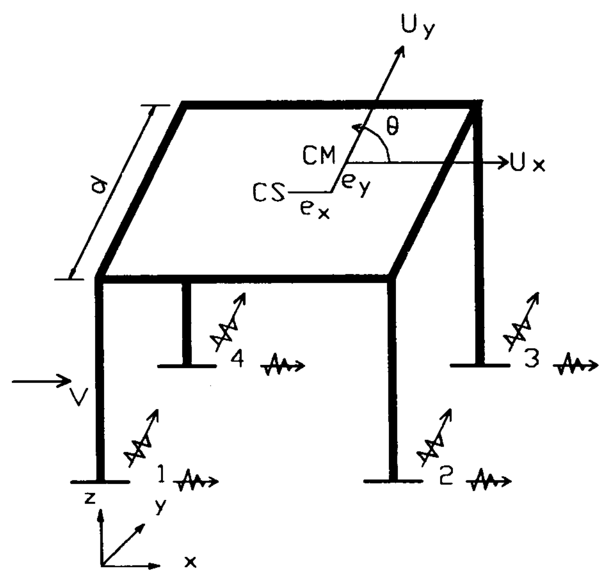

Fig. 1 Single storey multiple supported building model

STRUCTURAL MODEL AND EQUATIONS

The structural model considered in this study is the same as the one analyzed before with linear elastic assumption (Hao, 1998). For completeness of the current paper, it is briefly introduced here. It is a single storey, square rigid diaphragm, supported by four symmetric columns at the corners, as shown in Figure 1. It should be noted that the model does not intend to represent a single-storey building structure; rather it is an equivalent asymmetric system with multiple-supports used to study torsional responses of asymmetric system to spatially varying ground motions. The original stiffness center CS coincides with the geometric center, while the mass center CM is shifted away from the geometric center with eccentricities e_x and e_y in the *x*- and *y*-directions, respectively. The model has three structural degrees-of-freedom. They are the translational displacements in the *x*- and *y*-directions, u_x and u_y, and rotation about the mass center, θ. Because spatially varying seismic motions in both the *x*- and *y*-directions are considered, there are eight support displacements, $v_g = (v_{x1}\ v_{y1}\ v_{x2}\ v_{y2}\ v_{x3}\ v_{y3}\ v_{x4}\ v_{y4})^T$, as shown in Figure 1.

The total structural responses are the summation of the dynamic and quasi-static responses

$$u^t = u + u^{qs} \tag{1}$$

where $u^t = (u_x^t\ u_y^t\ \theta^t)$ is the total displacement vector. The quasi-static responses can be calculated by

$$u^{qs} = -K_{ss}^{-1}K_{sb}v_g \tag{2}$$

where K_{ss} is the stiffness matrix corresponding to the three structural degrees of freedom, and K_{sb} is the stiffness matrix corresponding to the coupled three structural degrees of freedom and eight support movements.

By lumping the column and floor masses to the mass center, and neglecting the quasi-static velocity-induced damping forces, which are zero if viscous damping is stiffness proportional and are very small for other types of damping, the dynamic response equations can be derived in matrix form as

$$m_{ss}\ddot{u} + C_{ss}\dot{u} + K_{ss}u = m_{ss}K_{ss}^{-1}K_{sb}\ddot{v}_g \tag{3}$$

where C_{ss} is the viscous damping matrix, and for the structural model shown in Figure 1,

$$m_{ss} = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{pmatrix} \tag{4}$$

$$K_{ss} = \begin{pmatrix} \sum_{i=1}^{4} k_i & 0 & -\sum_{i=1}^{4} k_i y_i \\ 0 & \sum_{i=1}^{4} k_i & \sum_{i=1}^{4} k_i x_i \\ -\sum_{i=1}^{4} k_i y_i & \sum_{i=1}^{4} k_i x_i & \sum_{i=1}^{4} k_i (x_i^2 + y_i^2) \end{pmatrix} \tag{5}$$

and

$$K_{sb} = \begin{pmatrix} -k_1 & 0 & -k_2 & 0 & -k_3 & 0 & -k_4 & 0 \\ 0 & -k_1 & 0 & -k_2 & 0 & -k_3 & 0 & -k_4 \\ y_1k_1 & -x_1k_1 & y_2k_2 & -x_2k_2 & y_3k_3 & -x_3k_3 & y_4k_4 & -x_4k_4 \end{pmatrix} \tag{6}$$

in which m is the lumped mass; I is the polar moment of inertia of the model about a vertical axis through the mass center; k_i is the *i*th column lateral stiffness; *d* is the building dimension; x_i and y_i are the co-ordinates of the *i*th column with the origin at CM; and $x_1 = x_4 = -d/2 - e_x$, $x_2 = x_3 = d/2 - e_x$, $y_1 = y_2 = -d/2 - e_y$, and $y_3 = y_4 = d/2 - e_y$.

Let K_t represent the lateral stiffness of the structure and K_θ, the torsional stiffness relative to the CM, where

$$K_t = \sum_{i=1}^{4} k_i \tag{7a}$$

$$K_\theta = \sum_{i=1}^{4} k_i (x_i^2 + y_i^2) \tag{7b}$$

Substituting Equations (5), (6) and (7) into Equation (2), the quasi-static displacements are:

$$u^{qs} = \begin{pmatrix} u_x^{qs} \\ u_y^{qs} \\ \theta^{qs} \end{pmatrix} = -\begin{pmatrix} \sum_{i=1}^{4} (-A_1 + A_3 \cdot y_i) k_i \cdot v_{xi} - (A_2 + A_3 \cdot x_i) k_i \cdot v_{yi} \\ \sum_{i=1}^{4} (-A_2 + A_5 \cdot y_i) k_i \cdot v_{xi} - (A_4 + A_5 \cdot x_i) k_i \cdot v_{yi} \\ \sum_{i=1}^{4} (-A_3 + A_6 \cdot y_i) k_i \cdot v_{xi} - (A_5 + A_6 \cdot x_i) k_i \cdot v_{yi} \end{pmatrix} \tag{8}$$

where

$$A_1 = \frac{(2d^2 + 4e_y^2) K_t^2 + 4d \cdot e_y \cdot K_t \cdot K_a - d^2 \cdot K_b^2}{d^2 \cdot K_t (2K_t^2 - K_a^2 - K_b^2)} \tag{9a}$$

$$A_2 = -\frac{(d \cdot K_a + 2e_y \cdot K_t) \cdot (d \cdot K_b + 2e_x \cdot K_t)}{d^2 \cdot K_t (2K_t^2 - K_a^2 - K_b^2)} \tag{9b}$$

$$A_3 = -\frac{4e_y \cdot K_t + 2d \cdot K_a}{d^2 \cdot (2K_t^2 - K_a^2 - K_b^2)} \tag{9c}$$

$$A_4 = \frac{(2d^2 + 4e_x^2) K_t^2 + 4d \cdot e_x \cdot K_t \cdot K_b - d^2 \cdot K_a^2}{d^2 \cdot K_t (2K_t^2 - K_a^2 - K_b^2)} \tag{9d}$$

$$A_5 = \frac{4e_x \cdot K_t + 2d \cdot K_b}{d^2 \cdot (2K_t^2 - K_a^2 - K_b^2)} \tag{9e}$$

$$A_6 = \frac{4K_t}{d^2 \cdot (2K_t^2 - K_a^2 - K_b^2)} \tag{9f}$$

$$K_a = k_1 + k_2 - k_3 - k_4 \tag{9g}$$

$$K_b = k_1 - k_2 - k_3 + k_4 \tag{9h}$$

From Figure 1, it can be derived that the base shear in the x-direction is

$$\begin{aligned} V_x &= \sum_{i=1}^{4} k_i (u_x^t - v_{xi} - y_i \cdot \theta^t) \\ &= k_1 (u_x^t - v_{x1}) + k_2 (u_x^t - v_{x2}) + k_3 (u_x^t - v_{x3}) + k_4 (u_x^t - v_{x4}) \\ &\quad + (k_1 + k_2)(\frac{d}{2} + e_y)\theta^t - (k_3 + k_4)(\frac{d}{2} - e_y)\theta^t \end{aligned} \tag{10}$$

Similarly, the base shear in the y-direction is

$$\begin{aligned} V_y &= \sum_{i=1}^{4} k_i (u_y^t - v_{yi} + x_i \cdot \theta^t) \\ &= k_1 (u_y^t - v_{y1}) + k_2 (u_y^t - v_{y2}) + k_3 (u_y^t - v_{y3}) + k_4 (u_y^t - v_{y4}) \\ &\quad - (k_1 + k_4)(\frac{d}{2} + e_x)\theta^t + (k_2 + k_3)(\frac{d}{2} - e_x)\theta^t \end{aligned} \tag{11}$$

The torque about the mass center is derived as

$$T=\sum_{i=1}^{4}k_i[-(u_x^t-v_{xi})\cdot y_i+(u_y^t-v_{yi})\cdot x_i+(x_i^2+y_i^2)\cdot\theta^t] \tag{12}$$

As can be noticed, the formulae derived above are considerably more complex than those given in the previous study (Hao, 1998). This is because in the non-linear analysis, owing to torsional response, the four columns will not yield simultaneously. On the other hand, in linear elastic analysis, both K_t and K_θ are constants.

If $K_t=4k, K_\theta=K_t(\frac{d^2}{2}+e_x^2+e_y^2)$, and $k_1=k_2=k_3=k_4=k$ are substituted into the above formulae, the formulae will be reduced to those presented before based on linear elastic assumption.

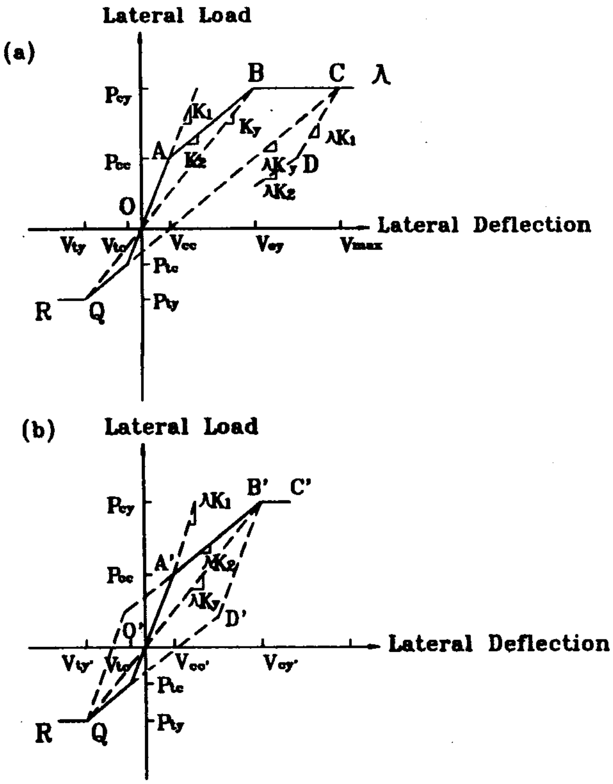

Fig. 2 Tri-linear Stiffness Degrading Hysteretic Model ($\lambda=\frac{V_{cy}}{V_{max}+V_{cy}}$)

It should also be noted that the base shear and torque now depend on total structural response, while they depend only on dynamic response in elastic analysis. This is because both base shear and torque depend on the relative displacement of each column as indicated in Equations (11) and (12). In elastic analysis, since the four columns have identical stiffness value, Equations (11) and (12) can be simplified to be only dynamic response-dependent (Clough and Penzien, 1993; Hao, 1997, 1998). In inelastic analysis, the four columns do not yield simultaneously because of nonuniform excitations and torsional response, the stiffnesses k_i of the four columns are not identical, and the above two equations can no longer be simplified.

A tri-linear stiffness degrading hysteretic model is used in the present inelastic analysis, as shown in Figure 2. This is modified from the model developed by others (Murakami and Penzien, 1975; Clough and Penzien, 1993). This model represents structural resistance behaviors of reinforced concrete in non-linear inelastic deformation and failure characteristics controlled primarily by flexure tension and compression. It is completely characterized by the following parameters, $K_1, K_2, K_y, P_{cc}, P_{cy}, V_{cy}, V_{cc}, P_{tc}, P_{ty}, V_{tc}$ and V_{ty}. Based on material properties of reinforced concrete, $P_{ty}=2.0P_{tc}$, $P_{cy}=2.0P_{cc}$, $V_{ty}=4.0V_{tc}$, $V_{cy}=4.0V_{cc}$, and $V_{cc}=6.0V_{tc}$ are used in the present study. The model is thus completely defined, if initial stiffness K_1 and compression cracking displacement V_{cc} are known. The initial stiffness K_1 can be determined from structural natural vibration period. The cracking displacement V_{cc} for RC structure is defined according to the NEHRP (1991) recommendation as explained in the following.

If the deformed shape of a moment-resisting reinforced concrete frame building is assumed to be linear, its yielding displacement Δ_y at two-thirds height of the building can be calculated as

$$\Delta_y = \frac{\Omega \cdot C_s \cdot g \cdot T^2}{(2\pi)^2} = \Omega \frac{1.2 A_v S}{R T^{2/3}} \cdot \frac{g \cdot T^2}{(2\pi)^2} \tag{13}$$

where Ω is the frame over-strength factor and is assumed to be 1.67 in the present study, C_s is the NEHRP base shear coefficient, g is the gravity acceleration, and T is the fundamental vibration period of the structure that can be estimated according to the NEHRP recommendation in terms of the building height and the frame type. In determining C_s, the coefficients $A_v = 0.4$, $S = 1.2$, and $R = 8$ are assumed.

The coefficients Ω, C_s, and the period T depend on the structural and material types and the building dynamic behaviors. Usually, their exact values cannot be obtained for a structure in the design process. For that reason, a conservative tensile-yielding displacement $V_{ty} = 2/3\ \Delta_y$ is used in the analysis, which is equivalent to the compressive cracking displacement $V_{cc} = \Delta_y$.

It should be noted that in seismic design, especially in performance-based design, besides shear force, the structural response ductility ratio is also an important design parameter. In the present study, however, only base shear and torque are presented in order for direct comparison with the results obtained before based on linear elastic analysis (Hao, 1998). Since both base shear and torque are given in the present paper, interested readers can have an approximate estimation of the ductility ratio, although it is not straightforward, especially when the four columns do not yield simultaneously.

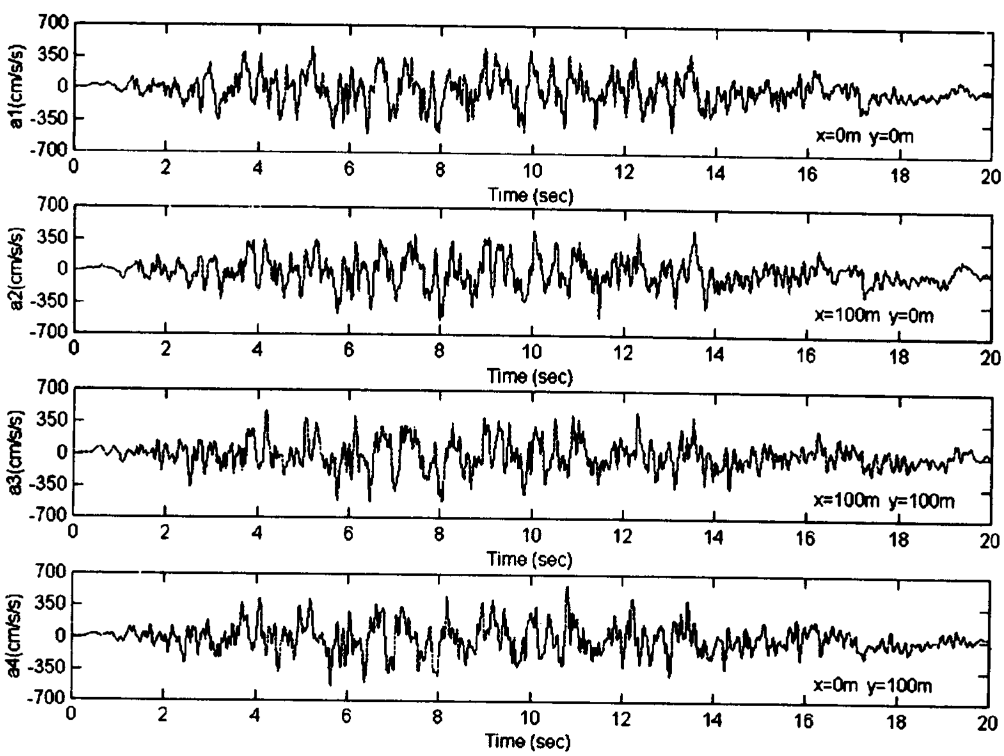

Fig. 3 Typical simulated spatially correlated ground accelerations in x – direction at the four structural supports

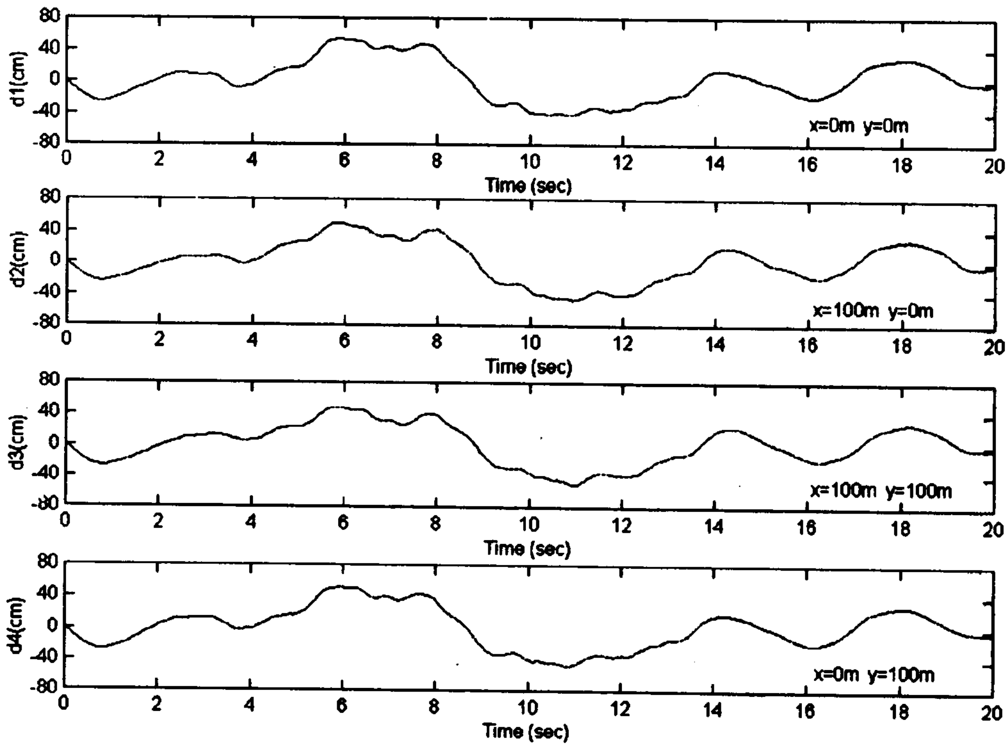

Fig. 4 Typical simulated spatially correlated ground displacements in x – direction at the four structural supports

SPATIAL SEISMIC GROUND MOTION

Similar to the previous study (Hao, 1998), 20 sets of spatially varying seismic ground motions are stochastically simulated and used as multiple inputs at the structural supports. The motions are simulated with a duration of 20.48 sec and a time increment of $dt = 0.01$ sec. The ground motion wave is assumed to be propagating in the x-direction with an apparent velocity $v = 1000$ m/s. Thus, the principal ground vibration direction coincides with the x-direction, and the ground motion components in the x- and y-directions are statistically independent (Penzien and Watabe, 1975). Therefore, these components are simulated independently.

Twenty sets of spatially correlated time histories are independently simulated to represent ground motions at the four structural supports in both the x- and y-directions. All the simulated time histories are iterated to be compatible with the empirical coherency loss function between each other, and are individually compatible with the Newmark and Hall (1982) response spectrum with 5% damping and normalized to 0.5g. Figure 3 shows a typical set of simulated ground accelerations in the x-direction at the four supports, and the corresponding displacements are shown in Figure 4. The coherency losses between the simulated accelerations at any two supports are also calculated and compared with the model function. Figure 5 shows a typical comparison. Figure 6 shows the response spectrum of a typical simulated time history and the Newmark-Hall spectrum. As can be seen, the simulated motions are compatible with the Newmark-Hall response spectrum individually, and with the coherency loss function between each other. More detailed information on the simulated ground motions and simulation technique can be found in references (Hao, 1989, 1998).

NUMERICAL EXAMPLES OF TORSIONAL RESPONSES

Linear elastic and non-linear inelastic responses are calculated in time domain by using the Newmark method with constant acceleration assumption in step-by-step integration. A 5% viscous damping is used for both the lateral and torsional modes. The 20 sets of independently simulated multiple ground motion time histories in both the x- and y-directions are used as input. Twenty independent calculations are carried out. As the standard deviations are much smaller (less than 10%) than the respective ensemble mean peak values obtained by using the results from the 20 time history analyses, only the mean peak values are presented and discussed.

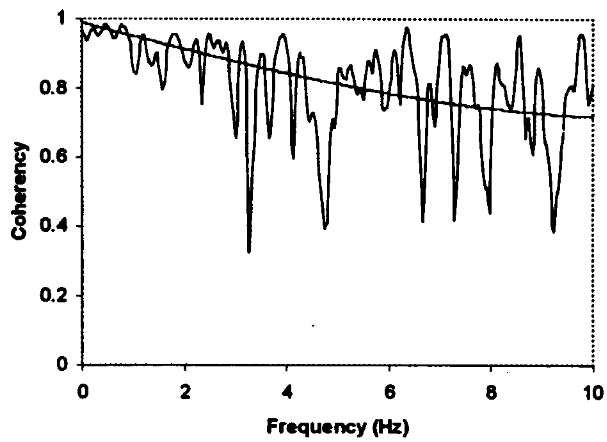

Fig. 5 Model coherency loss function and typical coherency loss between a_1 and a_2 with $d_1 = 100$ m and $d_t = 0$

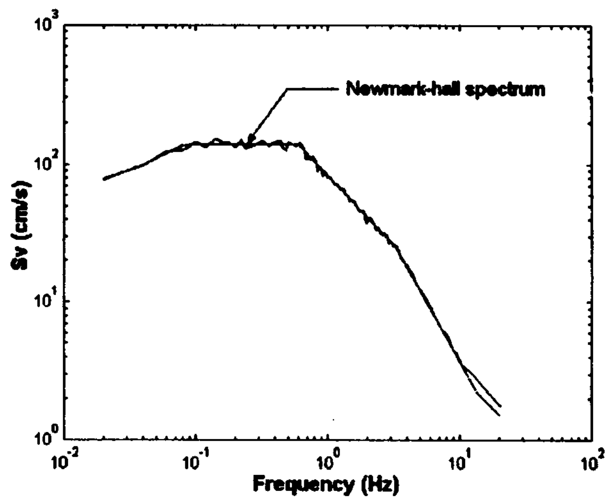

Fig. 6 Newmark-Hall response spectrum with 5% damping and normalized to 0.5g and the response spectrum of a typical simulated motion

In order to emphasize the coherency loss effect of non-uniform ground excitation on structure, the structural dimension of $d = 100$ m is used again in this study, as in a previous study considering only linear elastic response (Hao, 1998). It should be noted that this is only a virtual structural model. As it was explained, the reason to choose a very large span of 100 m is to emphasize the ground motion coherency loss effect. It has no physical meaning to structural vibration properties, as the vibration frequency of the structural model is a varying parameter in the calculation. Because the vibration frequency of the structural model or its stiffness is varied over a wide range in the parametric calculations, the numerical results presented cover ground motion phase shift effect on structures for most credible combinations of spatial ground motion, site and structure conditions. As ground motion coherency loss depends on many factors and is not well understood yet, using a large separation distance, such as $d = 100$ m, in multiple ground motion simulation will result in overestimation of the coherency loss effect on structural responses.

For comparison, the responses to uniform base excitation are also calculated by using the simulated motions at the support 1 as inputs to all supports. The uncoupled initial lateral vibration frequency

$f_x = \omega_x / 2\pi$ of the model is varied from 0.2 to 20 Hz in the calculation, where $\omega_x = \sqrt{K_t / m}$. This is achieved by continuously changing the mass of the model. Also, by varying the ratio of the lumped mass to the polar moment of inertia, three torsional stiffness cases are analyzed. They are torsionally flexible structure with an uncoupled torsional-to-lateral vibration frequency ratio $\omega_\theta / \omega_x = 0.75$, torsionally stiff structure with $\omega_\theta / \omega_x = 2.0$, and torsionally intermediate stiff structure with $\omega_\theta / \omega_x = 1.0$, in which $\omega_\theta = \sqrt{K_\theta / I}$ is the uncoupled torsional vibration frequency.

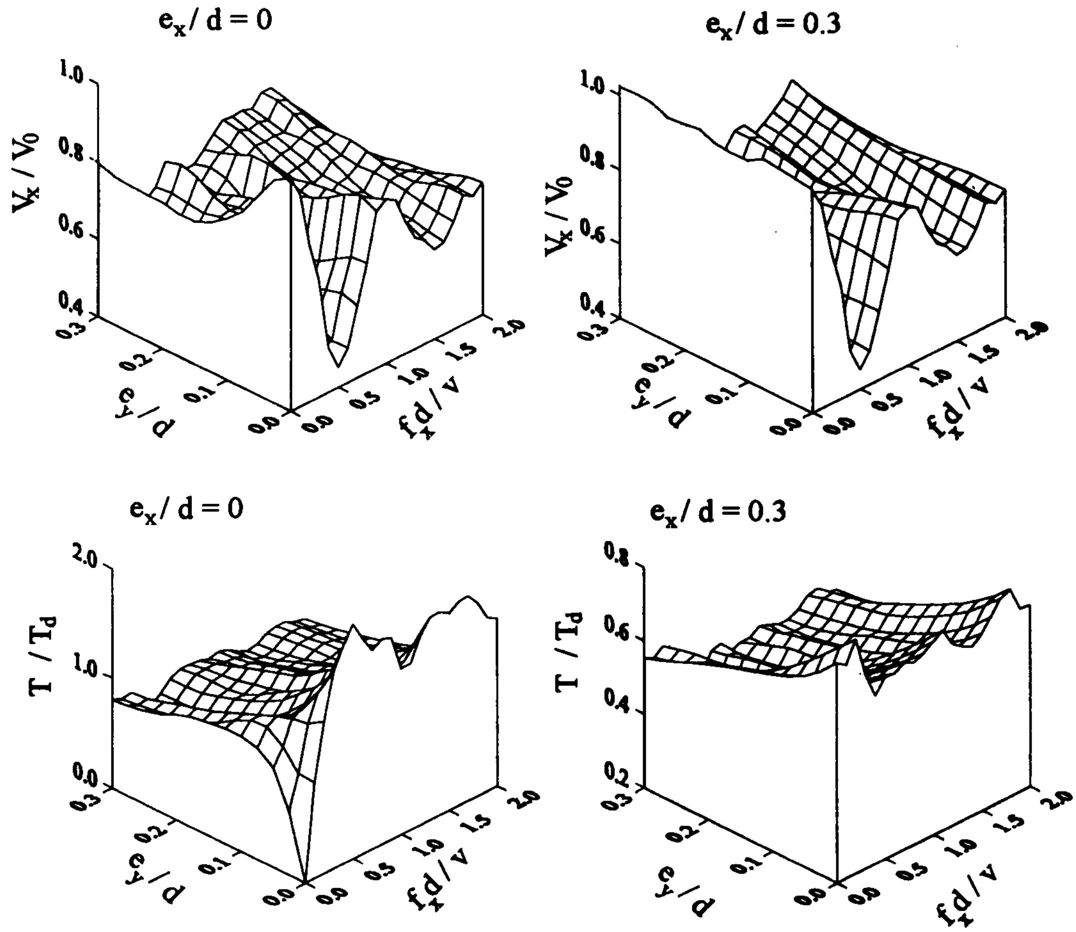

Fig. 7 Normalized linear elastic base shear and torque with different e_y $(\omega_\theta / \omega_x = 1.0)$

The eccentricity effect is investigated by varying the two eccentricities e_x and e_y from 0 to 30 m. This is achieved by varying the location of mass center. Numerical results are presented and discussed in terms of non-dimensional parameters $f_x d/v$, e_x/d and e_y/d. The parameter $f_x d/v$ measures the phase difference between the uncoupled lateral vibration mode and the ground motion phase shift, in which d/v is the time lag for ground motion to propagate over a distance d. $f_x d/v$ = 0.0 implies uniform ground excitation, which is equivalent to using d = 0.0 in multiple ground motion simulation; $f_x d/v$ = 0.5 and 1.5 imply out-of-phase condition between the lateral vibration mode and the dominant ground motion propagation phase shift; and $f_x d/v$ = 1.0 and 2.0 imply in-phase condition between them. In this study, the non-dimensional parameter $f_x d/v$ varies from 0.0 to 2.0, when f_x varies from 0.2 to 20 Hz and v = 1000 m/s. It should be noted that the lateral vibration frequency of concrete building can be approximately estimated by f_x = 50/h, as given in many seismic codes, in which h is the building height in meters (Paz, 1994). Then, $f_x d/v = 50d/(hv) = 50/(v\eta)$, where $\eta = h/d$

is the slenderness ratio of the building. It is obvious that the ratio of $f_x\, d/v$ = 0.0-2.0 covers a wide spectrum of realistic building structures for any credible combination of slenderness ratio η and ground motion apparent propagation velocity v.

For comparison purpose, linear elastic responses by using the initial stiffness of the non-linear stiffness model are also calculated. Figure 7 shows the normalized base shear in the x-direction and the normalized torque obtained by linear elastic analysis of the torsionally intermediate stiff structure with different eccentricities and vibration frequencies. The base shear V_x is normalized by V_{0x}, the base shear of a symmetric structure ($e_x/d = 0.0$ and $e_y/d = 0.0$) in the x-direction induced by uniform ground excitation. The torque is normalized by the design torque $T_d = V_{0x}.e_{xd} + V_{0y}.e_{yd}$, in which V_{0y} is the base shear of a symmetric structure in the y-direction induced by uniform ground motion, e_{xd} and e_{yd} are design eccentricities, defined as $e_{xd} = \alpha e_x + \beta d$ and $e_{yd} = \alpha e_y + \beta d$ where α is a coefficient for stiffness eccentricity and β is an accidental eccentricity coefficient. The accidental eccentricity is specified to cover the torsional responses induced by stiffness and mass eccentricity that is different from the one used in design when earthquake strikes, and it also implicitly accounts fŏr the torsional responses induced by torsional and spatially varying ground motions. Different codes give different α and β values. In the present study, α = 1.5 and β = 0.1 are used as specified, for example, in Mexico code (National University of Mexico, 1977) and Canadian code (NBCC, 1990).

Same observations as those reported in the previous study (Hao, 1998) can be drawn here. The normalized base shear and torque oscillate with the increase of dimensionless parameter $f_x\, d/v$. For a symmetric structure, the base shear reaches its maximum values at $f_x\, d/v$ = 1.0 and 2.0, and minimum values at $f_x\, d/v$ = 0.5 and 1.5, whereas the normalized torque reaches the peak values at $f_x\, d/v$ = 0.5 and 1.5, and the minimum values at $f_x\, d/v$ = 1.0 and 2.0. These indicate that when the ground motion phase shift is in-phase with the lateral vibration mode ($f_x\, d/v$ = 1.0 and 2.0), spatially varying ground motions produce the largest shear force, whereas they generate the smallest shear force when their phase shift is out-of-phase with the lateral vibration mode ($f_x\, d/v$ = 0.5 and 1.5). On the other hand, in-phase ground motion with the torsional vibration mode ($f_x\, d/v$ = 1.0 and 2.0) generates the smallest torque, while out-of-phase motion ($f_x\, d/v$ = 0.5 and 1.5) produces the largest torque. When the system is not symmetric, the responses still oscillate with $f_x\, d/v$, but the maximum and minimum values do not occur exactly at $f_x\, d/v$ = 0.5 and 1.5 or at 1.0 and 2.0. This is because coupled lateral and torsional vibration frequencies of the asymmetric system are not exactly equal to f_x and f_θ. Thus, the in-phase and out-of-phase excitations will not occur at those values.

In general, eccentricity and ground motion spatial variation cause reduction in base shear and induce torque. However, the base shear might increase slightly, when the system has significant eccentricities in both directions and is subjected to uniform excitation. The base shear is always reduced, when the system is subjected to spatially varying ground motions, indicating that the reduction of base shear by spatially varying ground motions is more significant than its increase due to the large two-way eccentricities.

The normalized torque is generally less than unity, when the system has significant eccentricities, implying the adequacy of the code torsional provisions. However, it is larger than unity when the system is symmetric or asymmetric with small stiffness eccentricities, indicating that the accidental torsional provision coefficient β might not be sufficient to cover the torque produced by spatially varying ground motions for torsionally intermediate stiff structures. Numerical results for other torsional stiffness cases ($\omega_\theta/\omega_x = 0.75$ and $\omega_\theta/\omega_x = 2.0$), which are not shown here, indicate that spatial ground motion effect is more pronounced in torsionally flexible structures, and less important in torsionally stiff structures. β = 0.1 is sufficient to cover the torsional responses of symmetric or small-eccentricity structures induced by spatial ground motion, if they are torsionally stiff; but this will substantially underestimate the torsional responses of torsionally flexible structures. However, it should be noted that this observation is based on results from the simplified structural model, which results in overestimation of the ground motion spatial variation effects on structures, especially when the structures have more supports, or site has one large mat foundation. More detailed analyses are deemed necessary to study the adequacy of code-specified accidental eccentricity coefficient.

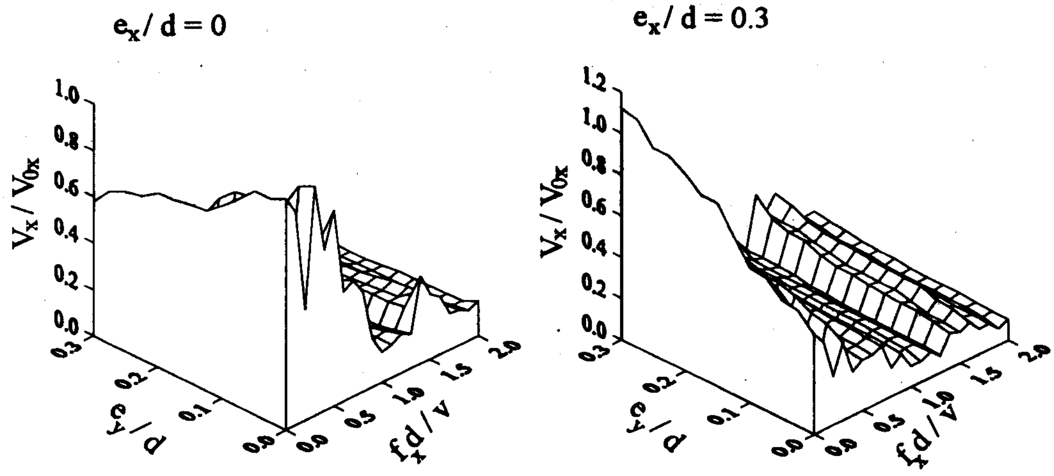

Fig. 8 Normalized non-linear inelastic base shear with different e_y $(\omega_\theta / \omega_x = 1.0)$

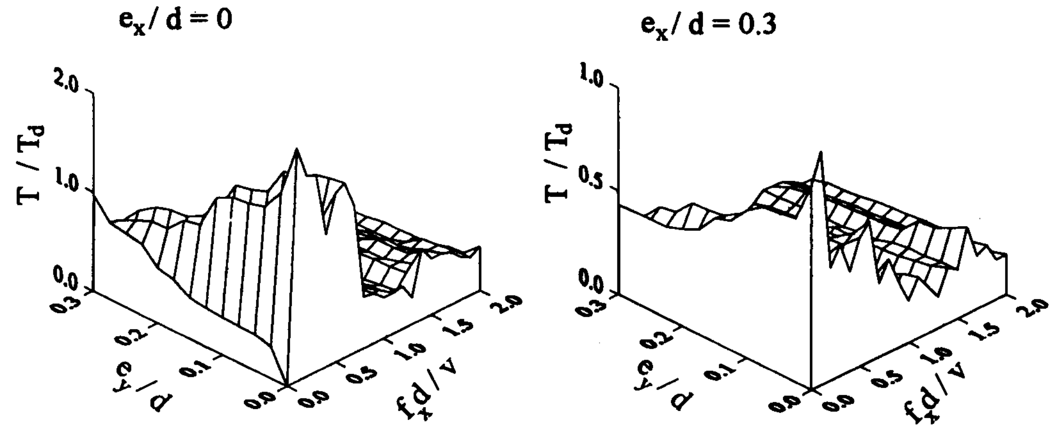

Fig. 9 Normalized non-linear inelastic torque with different e_y $(\omega_\theta / \omega_x = 1.0)$

Figure 8 shows the normalized base shear in the x-direction for the torsionally intermediate stiff structure. As shown, like those illustrated in Figure 7 for linear elastic analysis, both eccentricity and ground motion spatial variation generally reduce base shear, except when the system has significant eccentricities in both directions and is subjected to uniform excitation. Base shear in such cases, and the increment could be about 10%, when $e_x/d = e_y/d = 0.3$. The normalized base shear is always less than unity, when the system is subjected to spatially varying ground motions. These observations are similar to those described for linear elastic analysis. With the increase in parameter $f_x\, d/v$, the base shear oscillates and decreases in an overall trend. The oscillation, however, is not as prominent as in the elastic responses. The maximum and minimum shear forces do not necessarily occur at $f_x\, d/v$ = 0.5, 1.5 or 1.0 and 2.0 either. This is because of the degradation of stiffness in inelastic response which reduces the vibration frequency. Hence, in-phase and out-of-phase vibrations will not occur at the same $f_x\, d/v$ values.

The normalized base shear corresponding to the other two torsional stiffness cases, namely $\omega_\theta / \omega_x = 0.75$ and $\omega_\theta / \omega_x = 2.0$, are not shown here as they are very similar to those in Figure 8. This indicates that the normalized base shear is insensitive to the torsional stiffness. This observation is the same as that reported in a previous paper from linear elastic analysis.

Figure 9 shows the normalized torque of the torsionally intermediate stiff system. It shows again that the code torsional provision is sufficient, when the structure has large eccentricities or is subjected to uniform ground excitation. It is insufficient, when the system is symmetric or asymmetric with small eccentricities and is subjected to spatial ground motions. The out-of-phase and in-phase response is still observed here, but not exactly at $f_x\, d/v$ = 0.5, 1.5 or 1.0 and 2.0 due to yielding and eccentricities. When the structure has large eccentricities, spatially varying ground motions tend to produce smaller torque. Out-of-phase excitation is not observed, when the structure has large eccentricities. This is because of the coupling of the lateral and torsional modes, as well as due to the yielding owing to the increase in torsional responses for large eccentricities.

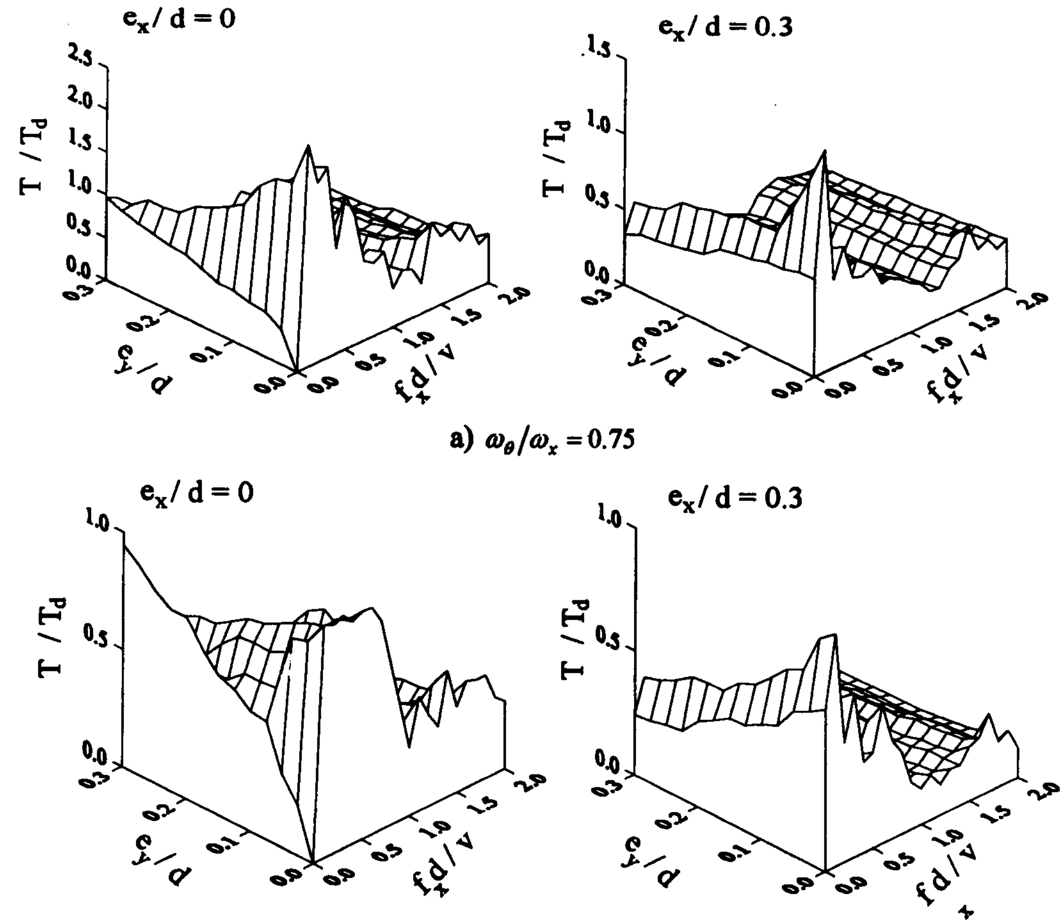

Fig. 10 Normalized non-linear inelastic torque with different e_y $(\omega_\theta/\omega_x = 0.75, 2.0)$

Figure 10 shows the normalized torque for the other two torsional stiffness cases. As shown, the normalized torque of the torsionally stiff case ($\omega_\theta / \omega_x = 2.0$) is always less than unity, even when the structure is symmetric and subjected to spatially varying ground motions. For the torsionally flexible case ($\omega_\theta / \omega_x = 0.75$), it is less than unity when the structure is under uniform excitation. But it is larger than unity, when the structure is symmetric or one-way eccentric and is subjected to spatially varying groun: motions with $f_x\, d/v$ less than 1.0. When the structure is two-way eccentric with $e_x/d = 0.3$, it is slightly larger than unity, only when e_y/d is small; otherwise, it is less than unity.

The present results are very similar to those obtained by linear elastic analysis (Hao, 1998), except that the effects of in-phase and out-of-phase excitations of spatial ground motion are not as significant, owing to stiffness degradation of the non-linear structure, as discussed above. They indicate that, no matter whether a torsional structure responds elastically or inelastically, current torsional provision of $\alpha = 1.5$ and $\beta = 0.1$ is sufficient for structures under uniform excitation. When a structure is under spatially varying ground motions, this is sufficient only when the structure is torsionally stiff. When the structure is torsionally intermediate stiff or torsionally flexible, this is adequate only when the structure has large eccentricities. However, this might not be sufficient when the structure is symmetric or asymmetric with small eccentricities, indicating that the code accidental torsional provision with $\beta = 0.1$ is not sufficient to cover the torsional responses induced by spatially varying ground motions.

EFFECTS OF SIMULTANEOUS BI-DIRECTIONAL GROUND MOTIONS

All of the above results are obtained by using simultaneous bi-directional ground motion inputs. These results indicate some increase in base shear force, when the structure has large two-way eccentricities, even if it is under uniform ground excitation. This implies that the assumed equivalence, as allowed in the design of lateral-load-resisting elements and ground motions in the x- and y-directions separately, might not be appropriate. To demonstrate this, some torsional responses of linear structures will be analyzed in the following by considering structural stiffness and ground motion in the x- and y-directions separately, and by comparing the results with those presented above.

The base shear in the x-direction subjected to bi-directional ground motion V_x is normalized by V_{xx}, the respective base shear obtained by considering ground motion in the x-direction only. Figure 11 shows the normalized base shear with varying e_y and $e_x/d = 0.0$ and 0.3 for the torsionally intermediate stiff structure ($\omega_\theta / \omega_x = 1.0$). As shown, when structure is symmetric or one-way eccentric ($e_x/d = 0.0$) and is subjected to uniform ground motion, V_x is equal to V_{xx}. If it is subjected to spatially varying ground excitation, V_x is equal to V_{xx} only when it is symmetric in the y-direction with $e_y/d = 0.0$. Otherwise, bi-directional ground motion will result in the increase of base shear. This effect is particularly significant, when the structure is subjected to non-uniform ground motion and has large eccentricities. Bi-directional and uni-directional uniform input results in the same shear force in the x-direction of the one-way eccentric structure with eccentricity in the y-direction, because the input motion in the y-direction is not coupled with the structural eccentricity in the same direction. Similarly, spatially varying input motion in the y-direction will not produce any shear force in the x-direction of a structure with $e_y/d = 0.0$ because of the uncoupling. Once the spatially varying ground motions are coupled with the structural eccentricity, ground motion in the y-direction will also produce some shear force in the x-direction. Thus, this results in the increase of shear force. The increment could be very significant, and is as much as 60% under the present conditions.

Figure 11 also shows the normalized mean torque of the structure by $T_0 = T_x + T_y$, where T_x and T_y are mean peak torques obtained by applying ground motion in the x- and y-directions separately. It shows that when the structure is one-way eccentric and is subjected to uniform ground motion, T is equal to T_0; otherwise, T is generally less than T_0. These observations can be explained by examining Equation (12). When structure is one-way eccentric with $e_x = 0.0$, and with linear elastic assumption, Equation (12) becomes $T = K_t[e_y u_x + (\frac{d^2}{2} + e_y^2)\theta]$, i.e., torque depends on lateral displacement u_x and rotation θ. Since uniform excitation in the y-direction will not produce any response u_x and θ, the normalized torque T/T_0 is unity. When structure is two-way eccentric and is subjected to uniform ground excitation, the normalized torque T/T_0 is always less than unity in the present example, indicating that the absolute summation of the largest T_x and T_y in the two directions overestimates the actual torque T. This is because T_x and T_y usually will not reach the maximum values at the same time instant.

Results for other two torsional stiffness cases, namely $\omega_\theta / \omega_x = 0.75$ and $\omega_\theta / \omega_x = 2.0$, are also calculated, but are not shown here as those are very similar to the ones illustrated in Figure 11. Results obtained by linear elastic analyses are not shown here either. They have the similar trends as shown in Figure 11, but appear more regular because of the constant stiffness of the four columns.

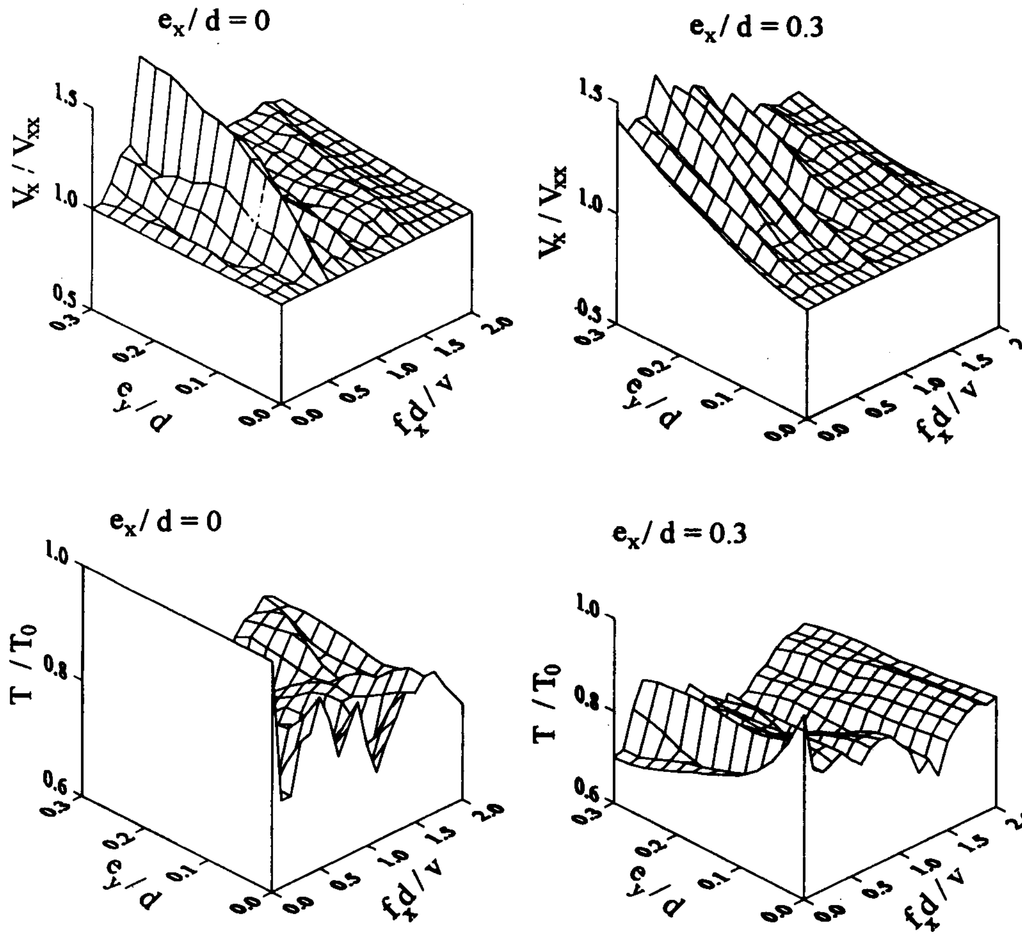

Fig. 11 Normalized base shear V_x/V_{xx} and torque T/T_0 with different e_y $(\omega_\theta/\omega_x=1.0)$

CONCLUSIONS

The responses of asymmetric building model subjected to two-directional horizontal non-uniform base excitations have been calculated by using non-linear analyses. The results have been presented and discussed in terms of non-dimensional parameters. It is found that the conclusions drawn before based on linear elastic analysis can still be made here, except that the effects of in-phase and out-of-phase excitations of spatial ground motion are less significant here owing to stiffness degradation of the non-linear structures.

It is also found that bi-directional ground motions will cause increase in base shear and might increase or decrease torque, especially when the structure has large eccentricities in both the *x*- and *y*-directions and is subjected to non-uniform ground motions.

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