

## **SIMULATION OF DESIGN RESPONSE SPECTRA OF STRONG-MOTION ACCELERATION IN KOYNA DAM REGION USING SEISMOLOGICAL SOURCE MODEL APPROACH**

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### **ABSTRACT**

A simple analytical approach based on Brune's omega-squared source model is presented to simulate various measures of strong earthquake ground motion, such as, RMS and peak ground motion amplitudes and the response spectra with different damping values in the Koyna dam region. Example results, computed to illustrate the applicability of the proposed approach, have shown very good consistency and agreement with the recorded ground motion.

### **INTRODUCTION**

Due to its simplicity and ability to simulate the characteristics of observed ground motion, Brune's (1970) source spectrum modified for path attenuation and site effects has been used in many past studies (e.g.; Boore, 1983; Faccioli, 1983; Boore and Joyner, 1984; Safak, 1988) for the simulation of strong earthquake ground motion. In the present study, using the results of Gupta and Trifunac (1988) on the order statistics of peaks of a random function of time, Brune's spectrum has been used to develop a stochastic model for computing the peak ground motion amplitudes and the response spectra with different damping values. The source parameters required to define the model are the seismic moment,  $M_0$ , and the stress drop,  $\Delta\sigma$ . The empirical correlations between the source parameters,  $M_0$  and  $\Delta\sigma$ , and the earthquake magnitude,  $M_L$ , developed by Gupta and Rambabu (1993) for the Koyna dam region have been used to simulate various measures of strong earthquake ground motion. Very good consistency and agreement have been found to exist between the simulated results and the

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recorded ground motion. Thus the proposed analytical approach can be used to simulate strong earthquake ground motion for given values of earthquake magnitude and source to site distance in the Koyna dam region.

### THE SOURCE MODEL

The seismological source model used here is based on the Brune's  $\omega$ -squared source spectrum (Brune, 1970), which has been used widely to explain the characteristics of high-frequency earthquake ground motion. According to Brune, the dislocation time function is directly related to the effective stress available to accelerate the two sides of a fault, and thus provides a physical basis for understanding the time function and the spectrum at high frequencies. According to this model, the Fourier amplitude spectrum of the displacement due to shear waves in the far-field can be idealised by a constant amplitude,  $\Omega_0$ , upto certain frequency,  $\omega_c$ , known as the corner frequency, and decays as  $\omega^{-2}$  after that. Thus the far-field acceleration spectrum of shear waves can be written as

$$A(\omega) = \Omega_0 \frac{\omega^2}{1 + (\omega/\omega_c)^2} \quad (1)$$

The low frequency spectral level,  $\Omega_0$ , is related to the seismic moment,  $M_0$ , as follows (Haskell, 1964)

$$\Omega_0 = \frac{M_0}{4\pi\rho R\beta^3} R_{\theta\phi} \quad (2)$$

where  $\beta$  is the shear-wave velocity,  $\rho$  is the density of the earth's crust,  $R$  is the hypocentral distance, and  $R_{\theta\phi}$  is the radiation pattern of the shear-waves. Thus the spectrum of eqn.(1) can be rewritten as

$$A(\omega) = \frac{M_0 R_{\theta\phi}}{4\pi\rho R\beta^3} \frac{\omega^2}{1 + (\omega/\omega_c)^2} \quad (3)$$

To account for the anelastic attenuation along the travel-path, amplitudes at all the frequencies are required to be scaled by  $\exp(-\omega R/2\beta Q)$ , where  $Q$  is the quality factor. Also, correcting for the free-surface amplification (factor  $F$ ) and vectorial partitioning of ground motion into two

horizontal components (factor P), the Fourier amplitude of a single horizontal component of acceleration at frequency  $\omega$  can be expressed as

$$A(\omega) = CM_0 S(\omega, \omega_c) e^{-\omega R/2\beta Q} / R \quad (4)$$

where C is a constant, given by,

$$C = \frac{R\theta\phi \cdot F \cdot P}{4\pi\rho\beta^3} \quad (5)$$

and

$$S(\omega, \omega_c) = \frac{\omega^2}{1 + (\omega/\omega_c)^2} \quad (6)$$

Following Thatcher and Hanks (1973), F is taken equal to 2, P equal to  $1/\sqrt{2}$  for equal partitioning of energy into two horizontal components, and the value of  $R\theta\phi$  is taken equal to 0.63, which is the r.m.s. value for the double couple radiation pattern.

In addition to the above, spectrum  $A(\omega)$  may also be required to be corrected for a sharp decrease in acceleration spectra with increasing frequency, usually observed beyond a cut-off frequency  $f_{max}$  (Hanks, 1982) and that for the site geologic condition (e.g.; Joyner and Fumel, 1984; Trifunac, 1990, etc.). However, these effects are not seen to be significant in the spectra of recorded accelerograms in the Koyna region (Gupta and Rambabu, 1993), and hence have been neglected for computing the example results in the present study.

From eqn.(4) it is seen that the spectra of different earthquakes can be described in terms of only two source parameters; viz., the seismic moment  $M_0$  and the corner frequency  $\omega_c$ . The corner frequency is further related to the effective stress-drop,  $\Delta\sigma$ , through the following expression (Gupta and Rambabu, 1993).

$$\omega_c = 98\pi\beta(\Delta\sigma/M_0)^{1/3}; \quad (7)$$

where  $\beta$  is in cm/sec,  $\Delta\sigma$  in bars, and  $M_0$  in dyne-cm. Using this expression into eqn.(4), the acceleration spectrum for specified values of stress-drop,  $\Delta\sigma$ , and moment,  $M_0$ , can be written as

$$A(\omega) = CK^2\omega^2 \frac{e^{-\omega R/2\beta Q}}{R} \frac{M_0(\Delta\sigma)^{2/3}}{\omega^2 M^{2/3} + K^2(\Delta\sigma)^{2/3}} \quad (8)$$

where  $K = 98\pi\beta$  is a constant.

The seismic moment is a measure of the size of an earthquake and it can be related through an empirical relation to earthquake magnitude (Hanks and Kanamori, 1979; Hanks and Boore, 1984; etc.). The corner frequency, or equivalently the stress drop, defines the high frequency amplitudes of an earthquake spectrum. Thus, unlike empirical scaling relations, the seismological source model describes only the low frequency amplitudes in terms of magnitude, and an additional source parameter; viz., the stress drop, has to be introduced to define the high frequency amplitudes of ground motion.

### THE STOCHASTIC THEORY

Gupta and Trifunac (1988) have developed the probability distributions to compute the expected, most probable or with a given confidence level the amplitudes of the peaks of various orders in a stationary random function of time,  $a(t)$ , from a knowledge of only the power spectral density function (PSDF),  $G(\omega)$ , of  $a(t)$ . But, in reality, the earthquake ground motion is nonstationary. Therefore, Gupta (1994) presented a method to define the strong motion stationary duration,  $T_s$ , of an accelerogram using which the Fourier amplitude spectrum,  $A(\omega)$ , of complete ground acceleration can be used to define  $G(\omega)$  corresponding to an equivalent stationary motion as follows :

$$G(\omega) = \frac{1}{\pi T_s} |A(\omega)|^2 \quad (9)$$

The probability distributions of Gupta and Trifunac (1988) are defined in terms of the root-mean-square amplitude,  $a_{rms}$ , of  $a(t)$ , total number of peaks,  $N$ , in  $a(t)$ , and a parameter  $\epsilon$  defining the bandwidth of the frequency spectrum of  $a(t)$ . These quantities are related to the zeroth, second and fourth order moments  $m_0$ ,  $m_2$  and  $m_4$  of the spectral density  $G(\omega)$  and the total duration,  $T$ , of  $a(t)$ . Thus from a knowledge of the stationary duration  $T_s$  and the total duration  $T$  alongwith the Fourier spectrum  $A(\omega)$  as computed from eqn.(8), the stochastic theory of Gupta and Trifunac (1988) can be used to compute the RMS and the peak acceleration amplitudes of the ground motion. The Fourier spectra of ground velocity and displacement time histories can be obtained by dividing  $A(\omega)$  with  $\omega$  and  $\omega^2$ , respectively. Using those in place of  $A(\omega)$ , the RMS and peak values of ground velocity and displacement amplitudes can also be evaluated.

Further, from the power spectral density function,  $G(\omega)$ , of ground acceleration the PSDF of the displacement response of a simple oscillator with natural frequency  $\omega_n$  and damping ratio  $\zeta$  can be defined as

$$E_n(\omega) = G(\omega)|H_n(\omega)|^2 \quad (10)$$

where  $H_n(\omega) = -(\omega_n^2 - \omega^2 + 2i\zeta\omega_n\omega)^{-1}$  is the complex-valued transfer function of the oscillator. From a knowledge of the power spectral density  $E_n(\omega)$ , the stochastic theory (Gupta and Trifunac, 1988) can be used to compute the expected amplitude of the highest peak (first-order) of the oscillator response, which by definition is the response spectrum amplitude at frequency  $\omega_n$ . By computing such amplitudes for different values of  $\omega_n$ , a complete response spectrum can be constructed for a given damping value.

## RESULTS AND DISCUSSION

The foregoing seismological source model approach has been used to simulate various measures of strong earthquake ground motion for the Koyna dam region, India and the results are compared with the corresponding values obtained from the analysis of recorded accelerograms. Because the horizontal bedrock motion in the far-field is generally dominated by the shear waves, eqn.(8) based on Brune's (1970) source model has been used to compute the Fourier spectrum of earthquake ground motion. This in turn has been used to define the power spectral density function of ground motion to evaluate the response spectra with different damping values from the stochastic approach mentioned above.

The seismic moment  $M_0$  and the stress drop  $\Delta\sigma$  for a given magnitude  $M_L$ , required to compute the example results, have been obtained from the empirical relations developed by Gupta and Rambabu (1993). They have developed the following least square regression relations for the seismic moment and the stress drop for the Koyna dam earthquakes.

$$\log_{10}M_0 = 1.018M_L + 17.597 \quad (11)$$

and

$$\Delta\sigma = 176.72\log_{10}M_0 - 3744.7 \quad (12)$$

The relation of eqn.(12) is valid only for  $0.3 \times 10^{22} \leq M_0 \leq 5.0 \times 10^{22}$ . For values of  $M_0$  greater than or smaller than this range, values of  $\Delta\sigma$  corresponding to the upper and the lower limits of  $M_0$  are to be used.

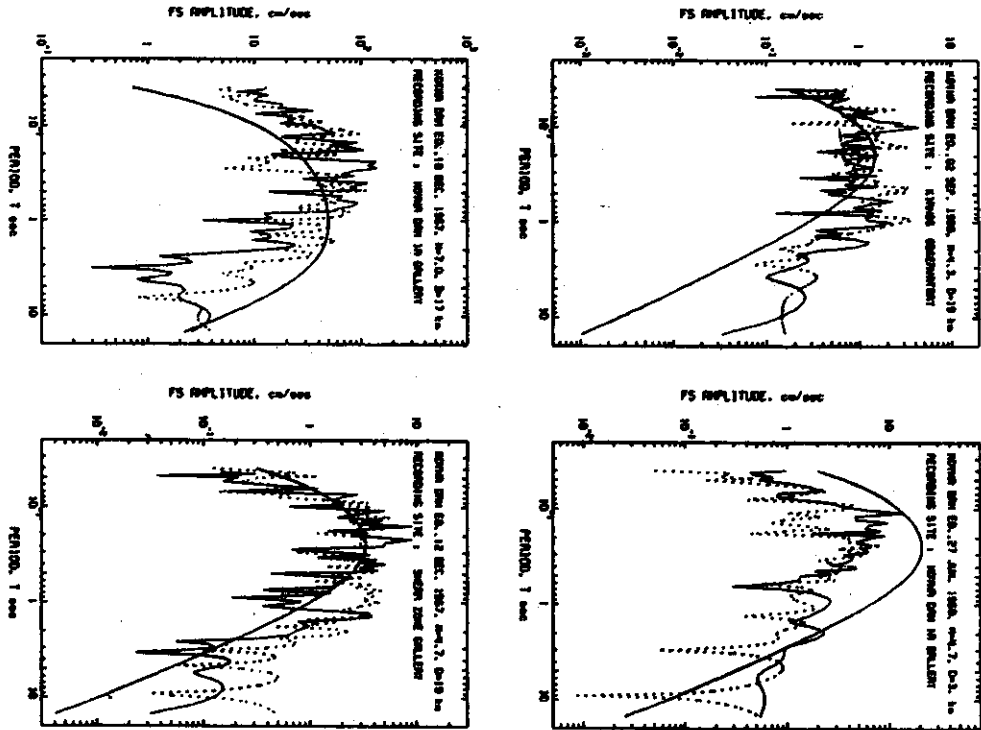


Fig. 1 : Comparison of the Fourier amplitude spectra computed from source model approach (smooth curves) with the spectra of two horizontal components (rough curves) for several real accelerograms recorded in the Koyna dam area.

The data base used by Gupta and Rambabu (1993) to develop the relation of eqn.(11) is dominated by earthquakes of magnitude below 4.5. Though this relation is quite consistent with the relations developed by other investigators for this magnitude range, it is not appropriate for magnitudes greater than about 5.0. Kanamori and Anderson (1975) have explained on theoretical basis that the coefficient of the magnitude term in relations of the form of eqn.(11) increases with increase in the magnitude range. Later, Hanks and Boore (1984) established by studying the behaviour of a large number of  $\log_{10}M_0 - M_L$  data from Central California as well as by theoretical computations that the  $\log_{10}M_0 - M_L$  relationship is characterized by a positive curvature. They have also advocated the idea that the  $\log_{10}M_0 - M_L$  relationships have very little regional dependence, if any. The behaviour of their data for intermediate magnitude range of about 3.5 to 6.5 is well approximated by the following mean regression relation due to Thatcher and Hanks (1973).

$$\log_{10}M_0 = 1.5M_L + 16.0 \quad (13)$$

The relation of eqn.(13) has been used to obtain the mean value of seismic moment for a given value of  $M_L$  in the present study.

Other parameters needed to evaluate  $A(\omega)$  from eqn.(8) are the shear-wave velocity  $\beta$ , density of the earth's crust  $\rho$ , quality factor  $Q$ , and the radiation pattern  $R_{\theta\phi}$ . Following Gupta and Rambabu(1993),  $\beta = 3.5 \times 10^6$  cm/sec,  $\rho = 2.8$  gm/cm<sup>3</sup>, and  $R_{\theta\phi} = 0.63$  have been used for the present computations. The high-frequency quality factor  $Q$  for an earthquake of magnitude  $M_L$  at a source to site distance of  $D$  in km has been obtained from the following relationship developed by Gupta and Rambabu (1994) for the Koyna dam area,

$$Q = 5.66D \quad (14)$$

With the above choice of the model parameters, agreement between the Fourier spectra of real accelerograms and the corresponding theoretical predictions has been studied for several accelerograms recorded in the Koyna region. A few representative examples for earthquakes with different magnitudes and hypocentral distances are shown in Fig.1. From the results in Fig.1, the matching between the theoretical and the actual spectra is seen to vary widely from case to case. For different examples, the matching is not equally good for the entire range of the wave-period. These variations in matching can be attributed to the fact that Brune's simple source

model used in this study is based on the average dislocation and stress-drop over the entire fault plane and the detailed history of faulting has been ignored. Also, the assumption of far-field made by us is not strictly valid in some cases. Further, the actual values of magnitude and source-to-site distance for the recorded accelerogram may be somewhat different than the reported values used for the theoretical predictions. The attenuation effects of the source-to-site path are also not modelled in a very rigorous way. In addition, due to large uncertainties generally associated with the source, path, and site parameters for an earthquake and due to random nature of earthquake events in an area, even for apparently identical set of various governing parameters, the spectra of actual earthquakes are found to differ significantly. The Koyna accelerograms are associated with large digitization and baseline distortions (Gupta et al., 1993) due to which low and high frequency spectral amplitudes of actual earthquakes may be in some error. Notwithstanding all these facts, the proposed approach provides a very simple method to obtain quite good estimates of the Fourier amplitude spectra for earthquake engineering applications. In general, the results in Fig.1 indicate very good consistency and agreement between the theoretical and the observed spectra. The quality of matching for the present results is seen to be equally good or even better compared to the results of many other similar studies (e.g., Singh et al., 1989; Ghosh, 1992; etc.) for different parts of the world.

From a knowledge of stationary duration,  $T_s$ , the above Fourier amplitude spectra can be used to obtain the PSDF of the ground acceleration, which can be used to compute the RMS and peak ground accelerations and the response spectra of ground acceleration. To compute the example results in the present study, the stationary durations for the Koyna dam accelerograms have been taken as evaluated by Gupta(1994). Fig.2 shows the comparison of theoretical RMS acceleration values with the resultant of the root-mean-square values of two horizontal components of recorded accelerograms in the Koyna dam area. Though, in view of the wide scattering in the observed data the results in Fig.2 can be considered to agree quite well with the observed values, the theoretical results seem to be somewhat biased towards the higher side. This perhaps is due to the fact that the theoretical formulation is based on the assumption of stationarity, whereas the actual accelerograms of Koyna earthquakes are seen to exhibit very strong nonstationary nature. To improve upon the matching in Fig.2 it would be



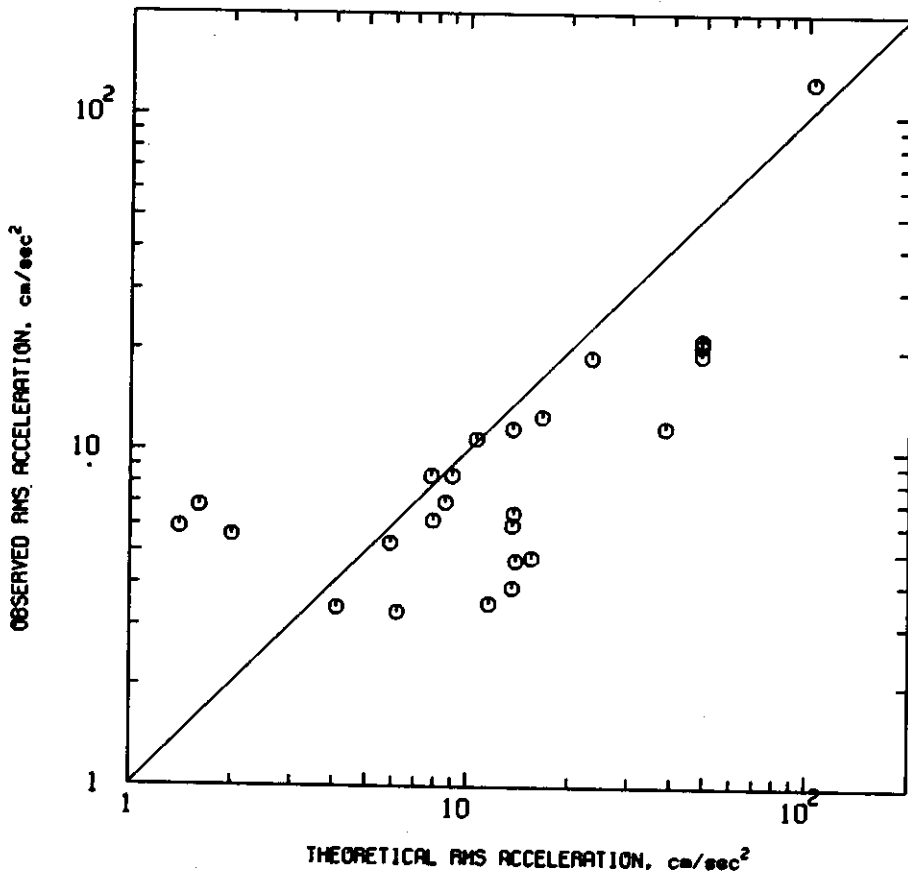


Fig.2 : Plot of the RMS acceleration computed from source model approach versus the observed values for the Koyna dam earthquakes.

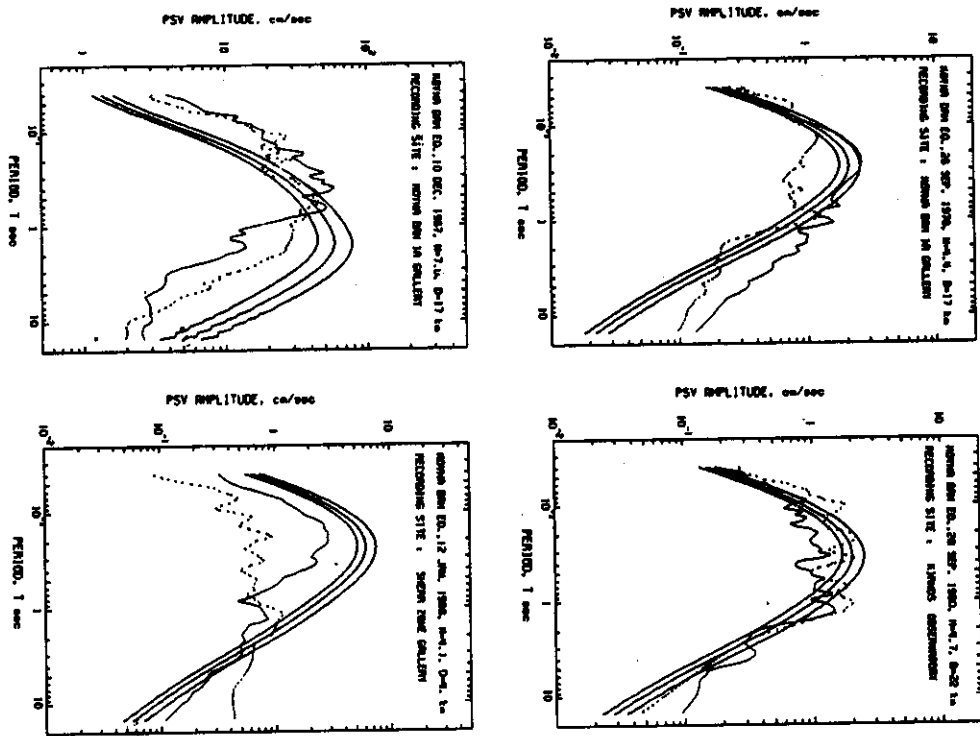


Fig.3 : Comparison of the simulated response spectra with a damping value of 5% of critical and confidence levels of 0.05, 0.50 and 0.95 (smooth curves) with the corresponding spectra of two horizontal components (rough curves) for several real accelerograms recorded in the Koyna dam area.

necessary to formulate an improved definition of the stationary duration,  $T_s$ , in eqn.(9). Peak acceleration is obtained by multiplying the RMS acceleration with an appropriate peak factor (Gupta and Trifunac, 1988). As the peak factor doesn't vary much for different accelerograms, the agreement between theoretical and actual peak acceleration values for Koyna dam region has been found to be similar to that for the RMS acceleration.

Fig.3 shows the 5% damped theoretical response spectra with confidence levels of 0.05, 0.50 and 0.95 compared with the spectra of two horizontal components of several recorded accelerograms in the Koyna area. It is observed that the response spectra computed by the seismological source approach are in good agreement with the corresponding spectra of the real accelerograms. Spectra for other damping values also showed similar agreement. The variation in the quality of matching for the example results in Fig.3 are again due to the various reasons as described for the cases of Fourier spectrum and RMS acceleration in Figs.1 and 2, respectively. The present results are simulated directly from a knowledge of the source parameters (seismic moment and stress-drop) and idealized attenuation characteristics along the propagation path. Unlike empirical attenuation relations developed by regression analysis of actually recorded data (Gupta and Rambahu, 1996), the theoretical results in Figs.1 to 3 are not really calibrated with the recorded ground motion. Thus the goodness of fit in the present study refers to the visual inspection only and no numerical measure can be assigned to the matching in a statistical sense.

Though rigorous fault kinematic models exist (Aki and Richards, 1980; Boore and Zoback, 1974; Hartzell and Heaton, 1986) to simulate very accurate strong ground motion amplitudes, those are not in common use for earthquake engineering applications. At present, the application of detailed fault rupture models is mostly limited to solving the inverse problem of finding the distribution of slip-rates, rupture-velocity and rise-time etc. over the entire fault plane during a past earthquake. For engineering applications one has to predict the design spectra for a postulated future earthquake, for which it is not possible to define all the details of the rupture process unequivocally.

## CONCLUSIONS

Based on the simple dislocation model of Brune (1970) for earthquake

source, an analytical approach has been presented to compute the far-field Fourier amplitude spectra of horizontal earthquake ground motion in the Koyna dam region. Using the results of the stochastic theory by Gupta and Trifunac (1988), these Fourier amplitude spectra have been used to simulate various measures of strong ground motion; viz. RMS and peak ground motion amplitudes and the response spectra with different damping values. Very good agreement and consistency of the theoretical ground motion amplitudes with the corresponding recorded data in the Koyna dam area, India shows that the presented formulation provides a simple and effective way to simulate design earthquake ground motion for earthquake engineering applications.

### ACKNOWLEDGEMENTS

The author is grateful to Mr. B.M. Rame Gowda, Joint Director, CW-PRS, Pune for providing all the help and encouragement during the course of the present research.

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