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DYNAMIC ANALYSIS OF BRIDGE SUBSTRUCTURE FOR EARTHQUAKE MOTION CONSIDERING SOIL STRUCTURE INTERACTION

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Introduction

The failure of the substructure is one of the major causes of damage or collapse of a bridge during the earthquake. The necessity of analysing the structural response considering the characteristics of ground motion, dynamic characteristics of structure and soil structure interaction is of paramount importance for safe and economic design in seismic areas.

Bridges particularly with long spans and high rise substructures situated in seismic zones IV and V of India are suscepitable to damage due to vibration effects of earthquake. Such bridges are required to be designed on the basis of dynamic consideration to have safe performance during the earthquake. The dynamic response analysis enables the determination of shears, moments and displacements in the structure considering the dynamic effect of loading and soil-structure interaction, which is not feasible with the equivalent static type of approach.

The purpose of this paper is to present the dynamic analysis of substructure of girder bridge using modal method and elastic response spectrum of earthquake. The embedded well foundation is represented by coupled foundation soil springs at the centre of gravity of rigid block on the basis of elastic half space solutions. The energy dissipation in the foundation medium is also considered through equivalent viscous damping. The influence of embedment depth and variation in soil properties on the dynamic response and foundation damping is studied. The influence of neglecting cross terms in foundation springs and contribution of the response in different modes is also studied. The important conclusions deduced from this investigation include that the structural response depends considerably on the stiffness of soil and depth of embedment. The damping ratio is seen to increase with the higher modes of vibration and softening of soils.

Mathematical Model

In this investigation only the girder bridges with spans simply supported on piers shall be covered. Each pier in such a bridge supports the ends of two spans one with a rocker end and other with a roller end. A single pier in such a case can be isolated (Fig. 1a) for the purpose of dynamic analysis because the rollers would prevent interaction between spans. A mathematical model consisting of masses lumped at discrete points is made as shown in, (Fig. 1 b), in order to find the natural frequencies and mode sphapes of the system. The number of masses into which the substructure should be represented depends on the number of modes required to be found. Usually fifteen to twenty masses are adequate for determining three or four modes with the reasonable accuracy.

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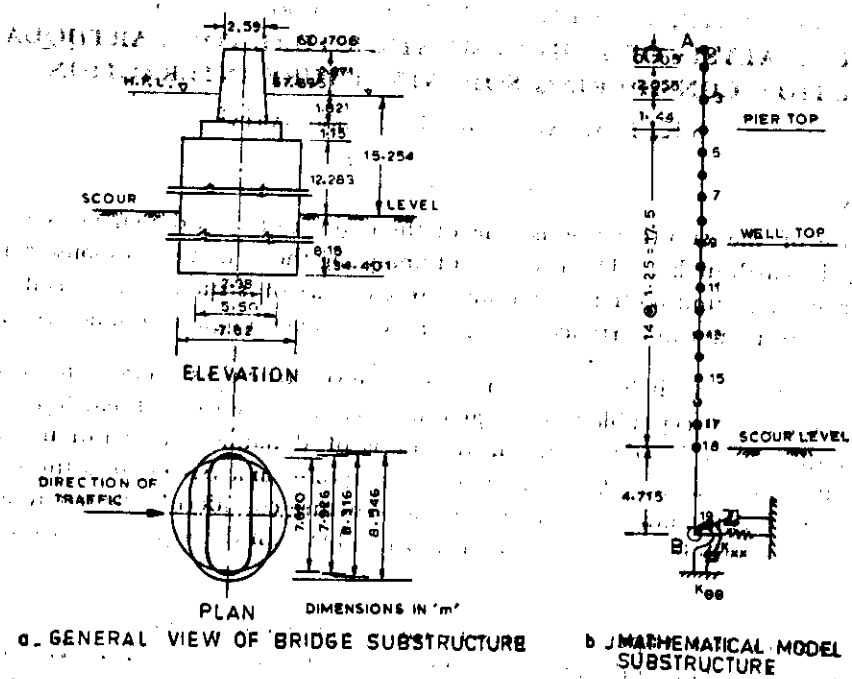

Fig. 1 Bridge substructure and its mathematical model

Idealization of foundation springs : The foundation is assumed to consist of elastic half space-base layer, surrounded by a elastic side layer providing the embedment. The structure below scour level is considered as rigid block. The resistance of base and side layer is replaced by coupled linear and rotational springs (Berdugo - Novak, 1972) at the centre of gravity of rigid block. The dissipation property of base and side layer are represented by coupled linear and rotational dashpots at the centre of gravity of rigid block, Fig. 2. The frequency independent foundation springs and dashpot matrix at the centre of gravity of rigid block (Novak, 1974) are given by.

$$[C] = \begin{bmatrix} C_{xx} & C_{x\theta} \\ C_{x\theta} & C_{\theta\theta} \end{bmatrix} \qquad [K] = \begin{bmatrix} K_{xx} & K_{x\theta} \\ K_{x\theta} & K_{\theta\theta} \end{bmatrix} \tag{1}$$

The spring and dashpot coefficients are given in Table 1. In this study the value of G for base layer is based on the assumed shear wave velocities for the soil properties. The side layer values, G_s are obtained from the relation (Berdugo-Novak, 1972)

$$\frac{G_s}{G} = \left(\frac{\rho_s}{\rho}\right)^{\theta} \tag{2}$$

In which ρ_s and ρ are mass density of side and base layer respectively.

Equivalent damping ratio due to foundation damping

The work done by the damping forces is given by.

Table 1

Spring and Dashpot Coefficients for Foundation Model

Translational Spring Contant K	$G_r\left(5.1+4.1\frac{G_s}{G}\delta\right)$
Cross-spring constant $K_{x\theta}=K_{\theta x}$	$-G_r\left(5.1\,Z_c+4.1\frac{G_s}{G}\delta\right)\left(Z_c-\frac{l}{2}\right)$
Rotational spring constant $K_{\theta\theta}$	$G_r^3\left[2.5+5.1\left(\frac{Z_c}{r}\right)^2+2.5\frac{G_s}{G}\delta+4.1\frac{G_s}{G}\delta\left\{\frac{\delta^2}{3}+\left(\frac{Z_c}{r}\right)^2-\delta\frac{Z_c}{r}\right\}\right]$
Cross-dashpot term $C_{x\theta}$	$-10.6\,(\rho G)^{0.5}\,r^2\left\{3.15\,Z_c+\delta\left(\frac{\rho_s G_s}{\rho G}\right)^{0.5}\left(Z_c-\frac{l}{2}\right)\right\}$
Rotational dashpot term $C_{\theta\theta}$	$(\rho G)^{0.5}\,r^4\left[0.43+3.15\frac{Z_c^2}{r^2}+10.68\,\delta\left(\frac{\rho_s G_s}{\rho G}\right)^{0.5}\left\{1.8+\left(\frac{\delta^2}{3}+\frac{Z_c^2}{r^2}-\frac{\delta Z_c}{r}\right)\right\}\right]$

Notations in Table 1 :

l = depth of embedment

r = base radius

$\delta = l/r$, embedment ratio

Z_c = distance of centre of gravity of foundation from base

G = shear modulus of base layer

G_s = Shear modulus of side layer

ρ_s = mass density of side layer

ρ = mass density of base layer

$$W=\int_0^T P(\dot{y})\,dy(t) \tag{3}$$

in which $P(\dot{y})$ = damping forces and T = time period. The damping ratio due to geometric damping in jth mode, ζ_j may them be defined as $\frac{1}{4\pi}$ times the ratio of the total work done (W) by the damping forces and moments during harmonic motion to the maximum energy (E_m) stored in the structure,

$$\zeta_j=\frac{W}{4\pi E_m}=\frac{1}{2\,p_j\,M_j}\left(C_{xx}\,u_{fj}+C_{\theta\theta}\,\theta_{fj}+2\,C_{x\theta}\,u_{fj}\,\theta_{fj}\right) \tag{4}$$

where,

E_m = maximum energy in the structure

p_j = jth circular frequency

M_j = generalised mass $= \sum_{i=1}^{n} (m_i u_{ij}^2 + I_i \theta_{ij}^2)$

m_i = ith mass point

I_i = mass moment of inertia of ith lumped mass about the horizontal axis.

u_{fj}, θ_{fj} = modal displacements of the foundation base in the jth mode.

Structures own damping ratio is however assumed to be 5 percent in all modes of vibration. Hence in a particular mode, total damping ratio,

$$\zeta_{jt} = \zeta_j + 0.05 \tag{5}$$

In this investigation the maximum value of ζ_{jt} was restricted to 20 percent.

Dynamic characteristics of structures

The dynamic characteristics of the structure, that is, natural frequencies and modes can be found by standard stiffness or transfer matrix procedures. Here the method of transfer functions is employed which is described in detail in reference (Arya and Thakkar, 1970). This method will be described only in brief here for completeness. The following transfer equations relate the V, M, Y, θ at Section $n-1$ to the section n considering bending, shear and rotatory inertia of the vibrating beam member. Figure 3 shows positive co-ordinates for the beam element.

$$\begin{aligned}
V_n &= V_{n-1} + m_{n-1}\, p^2\, Y_{n-1} \\
M_n &= M_{n-1} + V_n h_n - \rho I_n h_n p^2 \theta_{n-1} \\
\theta_n &= \theta_{n-1} + \frac{h_n}{2 E I_n} (M_n + M_{n-1}) \\
Y_n &= Y_{n-1} + h_n \theta_{n-1} + \frac{h_n^2}{3 E I_n}\left(M_{n-1} + \frac{M_n}{2}\right) - \frac{\sigma V_n h_n}{G A_n}
\end{aligned} \tag{6}$$

where,

V_n = Shear force in the nth segment

M_n, θ_n, Y_n = Bending moment, slope and deflection at nth mass point

h_n = length of nth segment

I_n = moment of inertia of member section in nth segment

E = modulus of elasticity

G = modulus of rigidity

ρ = mass density

σ = shape factor for shear deflection

These equations are successively applied to the segments and division points from one end to the other to yield the desired transfer functions. Now, out of the four boundary conditions, Shear and moment are zero at free end. The transfer functions are computed for two conditions, that is, (i) $Y_o = 1$, $\theta_o = 0$, and (ii) $Y_o = 0$, $\theta_o = 1$ at the free end. The coefficient matrix (D) is computed at the partially fixed end B in these two steps to relate the quantities at the end B with those of end A(Fig. 1b)

$$\begin{bmatrix} Y \\ \theta \\ M \\ V \end{bmatrix}_B = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \\ d_{31} & d_{32} \\ d_{41} & d_{42} \end{bmatrix} \begin{bmatrix} Y \\ \theta \end{bmatrix}_A \tag{7}$$

The shear and moment are related with deflection and slope at the partially fixed end by the following relation

$$\begin{bmatrix} V \\ M \end{bmatrix}_B = \begin{bmatrix} K_{xx} & K_{x\theta} \\ K_{x\theta} & K_{\theta\theta} \end{bmatrix} \begin{bmatrix} Y \\ -\theta \end{bmatrix}_B \tag{8}$$

The negative sign in θ is used because sign convention in transfer functions (Fig. 3), and stiffness coefficients (Fig. 2) is opposite to each other. Substituting the values of, V, M, Y, θ at B from (7) in (8), we get the following two equations.

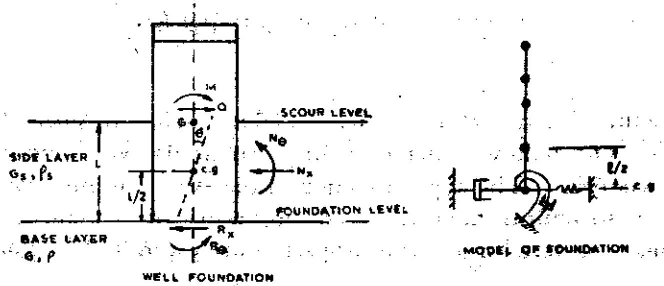

Fig. 2 Soil foundation model

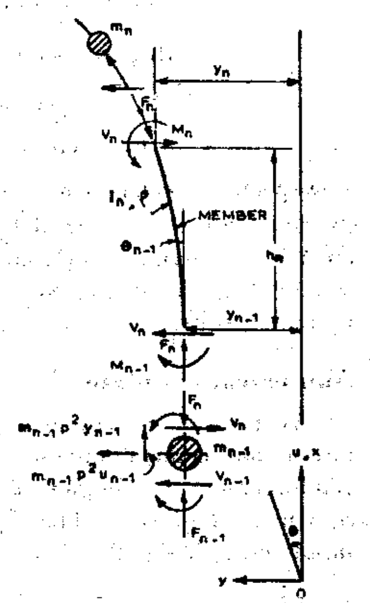

Fig. 3 Forces at the end of deformed straight elastic member

$$\begin{bmatrix} (d_{41} - K_{xx}\, d_{11} + K_{x\theta}\, d_{21}) & (d_{42} - K_{xx}\, d_{12} + K_{x\theta}\, d_{22}) \\ (d_{31} - K_{x\theta}\, d_{11} + K_{\theta\theta}\, d_{21}) & (d_{32} - K_{x\theta}\, d_{12} + K_{\theta\theta}\, d_{22}) \end{bmatrix} \begin{bmatrix} Y \\ \theta \end{bmatrix}_A = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{9}$$

For nontrivial solution for Y_A, θ_A, the determinant should vanish, the zero of the determinant will then give the frequency of the structure. The mode shape is calculated by a process of back substitution in transfer functions taking $Y_A = 1$ and θ_A calculated from one of the equations in (9).

Modal analysis

The dynamic response in rth mode of vibration at any section due to earthquake may be found from the expression,

$$X_r = C_r\, x_r\, S_{dr} \tag{10}$$

where,

X_r = dynamic response which may be shear, bending moment, deflection or any other quantity.

x_r = modal value of the response under consideration

C_r = mode participation factor

$$= \frac{\sum_{i=1}^{N} m_i Y_{ir}}{\sum_{i=1}^{N} m_i Y_{ir}^2}$$

m_i = ith mass point

Y_{ir} = modal deflection at ith mass point in rth mode

S_{dr} = spectral di placement in rth mode $\left(\frac{S_{ar}}{p_r^2}\right)$

S_{ar} = spectral acceleration in rth mode

p_r = circular frequency in rth mode

The spectral acceleration depends upon earthquake motion, time period and damping of the structure and is obtained by normalisation of the response spectra curves for 40 earthquake records at alluvial sites (Khanna, et al. 1977), the peak design ground acceleration is assumed to be 0.20 g for the area under consideration. The total structural response is then obtained by taking squre root of sum of squares (S.R.S.S.) of individual modal responses.

Numerical example

A typical railway bridge structure having eight spans, 33.09 m each with steel truss girders and cement concrete solid piers and hollow concrete cylindrical wells with nominal reinforcement is adopted for the analysis. The typical section of pier-well combination is shown in Fig. 1a. The significant structural data material properties of structure are tabulated in Table 2. The variations considered in the soil properties are given in Table 3.

Results of dynamic analysis

(i) Period of the structure : The fundamental time period of the structure varies from T = 0.21 s for fixed base condition to, T = 0.81 s for soil type 3 (softest soil). These period were observed for minimum depth of embedment.

(ii) Mode shapes of substructure : The first mode of the structure is a rocking mode while the second mode is bending in the fixed base condition and translatory for other soil types (Fig. 4). The third mode is bending in nature but involves significant bending and translatory movements of foundation.

(iii) Effect of soil type on natural period and damping : In all embedment conditions, time period and damping in first mode increases with the decreasing stiffness of soil (Fig. 5). In higher modes, same tendency is observed except damping in second

Table 2

Significant Structural Data and Material Properties of Bridge

Bridge type 1	**Steel truss bridge with simply supported spans**
Spans	**8 × 33.09m**
Superstructure	**Steel truss spans resting on rocker and roller bearings**
Piers	**Single solid concrete (M 100) with nominal surface reinforcements**
Wells	**7.62m overall diameter, concrete (M 100) hollow wells**
Height from base to the top of pier	**26.305m**
Modulus of elasticity of substructure	1800000.0 t/m²
Poisson's ratio	0.15
Weight density of substructure	2.24 t/m³
Variations in	8.18m Minimum (Min)
depth of embedment	13.18m Average (Ave)
considered	19.43m Maximum (Max)

Table 3

Soil Properties and its Variations

Soil properties	Type of soils		
	1	2	3
Shear wave velocity of base layer (m/s)	300.0	200.0	200.0
Weight density of base layer (t/m³)	2.0	2.0	2 0
Weight density of side layer (t/m³)	1.60	2.0	1.60
Shear modulus of base layer (t/m²)	18349.0	8155.0	8155.0
Shear modulus of base layer (t/m²)	9395.0	8155 0	4175.0

mode which increases first and then reduces in soil type 3 and there is a gradual reduction in third mode in this case.

(iv) Effect of embedment (depth/base radius) : The time period decreases with the increased depth of embedment while damping increases considerably, (Fig. 6).

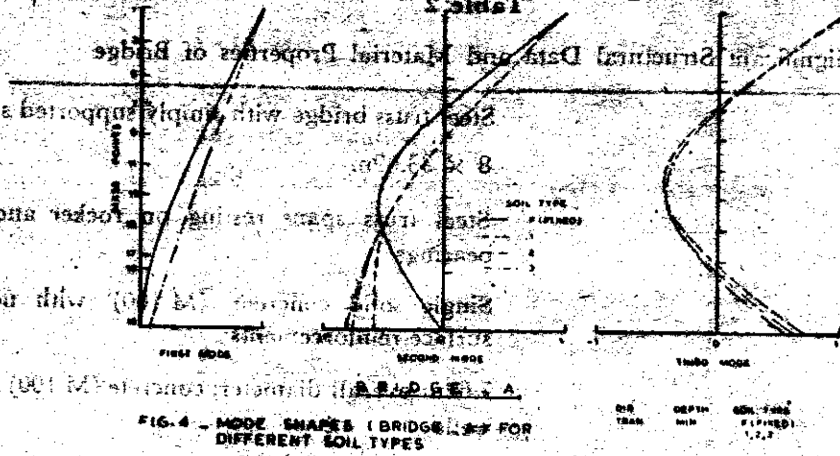

Fig. 4 Mode shapes (bridge - A) for different soil types

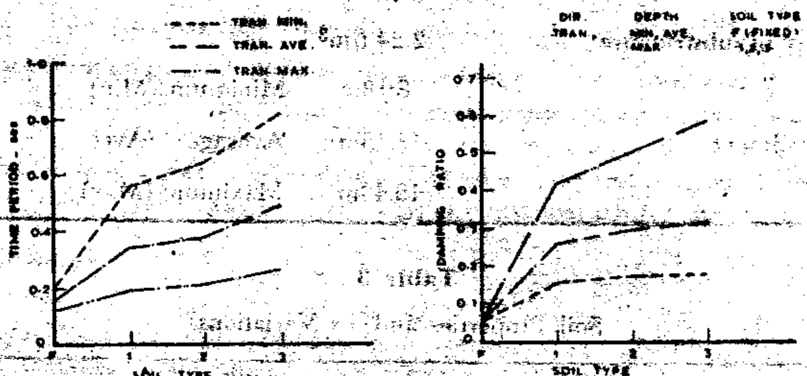

Fig. 5 Effect on first mode, time period and damping for different soil types

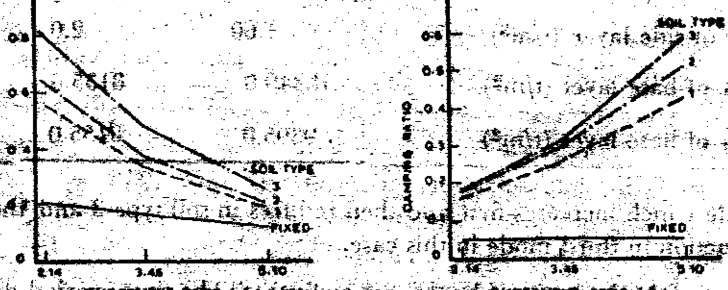

Fig. 6 Effect of embedment ratio on first mode, time period and damping

(v) Effect of soil type on dynamic response : Under all embedment depths shear force and bending moment decreases in a progressive manner with reduced strength of soil. The dynamic response values have greater variation near the base. The deflection is largest for soil type 3 while equivalent seismic coefficient is greatest for fixed base condition. Similar is the trend in the values of effective acceleration (Fig. 7).

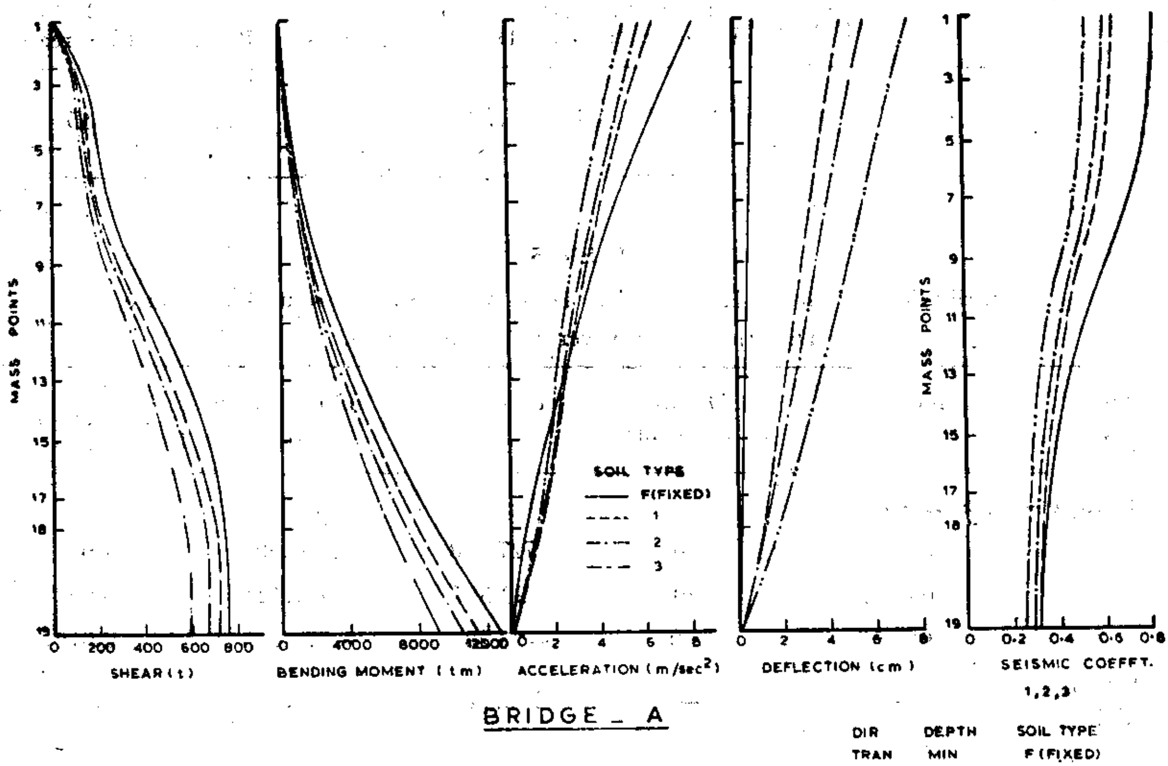

Fig. 7 Dynamic structural response (bridge-A) for different soil types

(vi) Effect of ommiting cross terms in spring/dashpot on dynamic response : In comparison with the normal case (cross terms included) the maximum structural response viz, shear and bending moment values increase if the cross terms are neglected. The variation is higher in soil type 3. The percentage increments in shear and moment and decrease in deflection are given in Table 4.

(vii) Contribution of individual modal responses : Table 5 shows the contributions of first second and third mode on shear, moment and deflection. It can be seen that the first mode response is dominant in most of the cases.

Conclusions

The following important conclusions are derived from the above study :

Table 4

Percentage Variation in Maximum Structural Response When Cross Terms are Neglected

Bridge	Depth	Direction	Percentage variation between normal case and when cross terms are neglected						
			Soil type 1				Soil type 3		
			Decrease	Shear Increase	Moment Increase	Defln. Decrease	Shear Increase	Moment Increase	Defln. Decrease
A	Min	Tran	—	103	104	29	111	112	27

Table 5

Contribution of Modal Response on Total Maximum Structural Response

Direction = Transverse, Depth = Min.

Mode No.	Time Period (secs)	Damping (percent)	Base Shear (t)	Base Moment (tm)	Top Deflection (cm)
Bridge - A, Soil - 1					
1	0.5603	10	727.738	11293.784	4.6
2	0.0951	20	92.527	467.42	0.05
3	0.0456	20	−23.28	−379.065	0.01
SRSS Value	—	—	733.44	11295.0	4.63
Bridge - A, Soil - 3					
1	0.8082	10	588.409	9066.725	7.60
2	0.1257	20	82.245	673.996	0.09
3	0.3505	20	−14.758	−23.56	0.002
SRSS Value	—	—	593.69	9088.2	7.602

1. The structural response is dependent considerebly on the stiffness of soil and depth of embedment, that is soil structure interaction significantly affects the dynamic response.
2. The fixed base conditions overestimates the shear force and bending moment and underestimates deflection.

3. The structural response is mostly contained in the first mode of vibration.
4. The effect of ommitting cross-terms in foundation stiffness and damping matrix on dynamic response is significant.

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