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PREDICTION OF DISPLACEMENTS OF RETAINING WALLS UNDER DYNAMIC CONDITIONS

by

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ABSTRACT

In this paper an attempt is made to develop a method of analysis for the estimation of displacements of rigid retaining walls under dynamic conditions incorporating combined rotation and translation by modelling the backfill as closely spaced independent elastic springs fixed to the back of retaining wall at different locations and the other ends of the springs are fixed to an immovable support. The mass of the wall is lumped at its centre of gravity. The backfill is treated as elasto-plastic material when the wall moves away from the backfill and elastic when wall moves towards the backfill.

Key words : Retaining Wall, Displacement, Earth Pressure. Dynamic Condition, Anlytical.

INTRODUCTION

Retaining walls are susceptible to failure if their displacements under static and dynamic conditions are not properly predicted. The method of designing retaining walls is incomplete without displacement criteria. The effect of displacement on the behaviour of a retaining wall under static condition is thoroughly investigated by the authors and complete details are given elsewhere (Reddy, 1985). The design of retaining walls based on allowable displacement under dynamic conditions has gained importance in recent years. There are very few methods available in the published literature to compute displacements of retaining walls during earthquakes namely, Richards-Elms Model (1974) based on Newmarks approach (1965), Solution in pure rotation (Prakash et al, 1981) and Nadim-Whitman Model (1983) using finite element technique. The details of these methods are presented elsewhere (Reddy, 1985).

In the present paper, a simple and sufficiently accurate method of dynamic analysis for the estimation of displacement of rigid retaining walls incorporating combined rotation and translation is proposed. The method

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is also able to compute earthforce and pressure distribution for any earthquake induced displacement of wall. It may be emphasised that the previous investigators obtained only the displacement and not the earth pressure and its distribution along the height of wall.

PROPOSED METHOD

When subjected to earthquake loading, the resulting motion of the retaining wall is complex. It may be reasonably idealized as consisting of translational motion or rotational motion or both depending upon the foundation conditions. But no method is available to predict displacement for rigid retaining walls incorporating combined rotation and translation, evaluating the individual contribution of rotational and translational motions towards its total displacement and taking into account the dynamic load, wall and soil parameters. In this paper, an attempt is made in this direction.

(i) **Mathematical Model:**—The Model of rigid retaining wall, backfill and foundation soil should be such that it results in translation and rotation simultaneously when subjected to earthquake loading. Therefore the model should have two degrees of freedom. In practice, cross-section of rigid retaining wall varies to a great extent. A reasonable approximation is, therefore, made by lumping the mass of the rigid retaining wall at its centre of gravity. The backfill soil is replaced by closely spaced independent elastic springs as shown in Fig. 1. To find the spring constants, soil modulii values have been used. In case of soil modulus varying linearly with depth (cohesionless soils), the soil reaction is assumed to act as a loading intensity. Treating this load to be acting on a beam of length equal to the height of retaining wall, the reactions at different points are evaluated treating this beam to be simply supported at these points. For the retaining wall of height H, divided into a convenient number of equal segments of height $\triangle h$, the reactions and hence the spring constants at various division points would be as under,

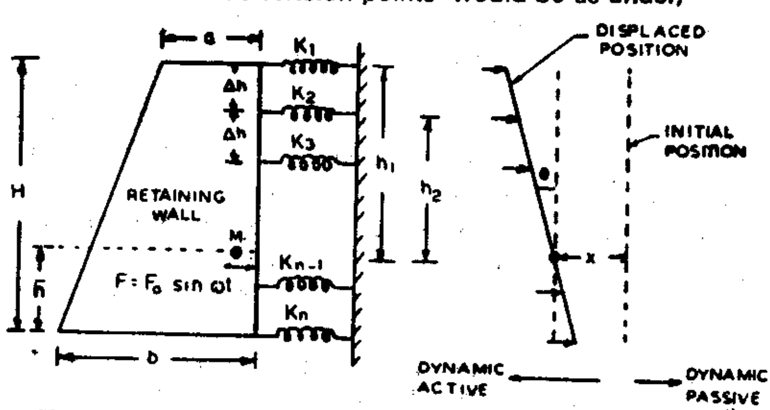

Fig. 1—Mathematical Model For Displacement Analysis Under Dynamic Condition

$$k_1 = 1/6\ n_h\ (\triangle h)^2$$
$$k_2 = n_h\ (\triangle h)^2$$
$$k_3 = 2\ n_h\ (\triangle h)^2$$
$$\vdots$$
$$k_i = (i-1)\ n_h\ (\triangle h)^2$$
$$\vdots$$
$$k_n = 1/6\ (3n-1)\ n_h\ (\triangle h)^2$$

where n_h is the constant of horizontal subgrade reaction having different values in active and passive conditions, k_1 and k_n are the spring constants at the top most and bottom most points. k_i is the spring constant at any division point 'i'.

(ii) **Assumptions :-** The proposed method is based on the following assumptions :

1. The earthquake motion may be considered as an equivalent sinusoidal motion with uniform peak acceleration and the total displacement equal to residual displacement per cycle multiplied by the number of cycles.

2. Soil stiffenesses (or spring constants) for displacement of wall towards the backfill and away from the backfill are different.

3. Soil participating in vibration, damping of soil and base friction are neglected

Assumptions 1 and 2 are usually made in such an analysis while assumption 3 needs justification.

It is difficult to determine analytically the soil mass that would participate in vibrations alongwith wall when the undergoes translational and rotational motions simultaneously. Neglecting this mass, the method gives higher displacements and the solution is conservative. However the mass of vibrating soil can be found out by carefully conducted experiments. For the case of pure translation, Nandakumaran (1973) has conducted experiments to determine the vibrating soil mass and concluded that it can be taken equal to 0.8 times the mass of Rankine's wedge. By adopting similar technique, the soil mass vibrating alongwith rigid retaining wall under combined rotational and translational motions can be found out Then it is added to the mass of the wall to lump at centre of gravity and the analysis can be carried out without any changes.

In soils it is customary to consider values of damping such as 15% or 20% of critical in view of larger energy absorption compared to other engineering structural materials. In the present analysis however, energy absorption in the form of plastic displacement of the wall has been considered. Therefore smaller damping values would be appropriate. Neglecting even this smaller damping, the displacement of the wall by this method will be more than the actual displacement.

The displacement of retaining wall is greatly influenced by base friction. In case of walls in alluvial deposits and at the waterfront, translational motion is predominant In some other cases, the walls may have predominant rotational motion. But in general, for any type of foundation soil, retaining wall possesses translational and rotational motions simultaneously. For rigid retaining walls, the stab lity is mainly due to its gravity, hence base friction contribution is significant. However, neglecting the base friction, the analysis will lead to an overestimation of the displacement.

The authors are engaged in research to refine the proposed model by including vibrating soil mass, damping of soil and base friction so that the analysis can predict displacements close to the actual displacements.

(iii) **Analysis**:-To study the response characteristics of the system, two cases are considered, one in which plastic deformations do not take place (elastic system) and the other in which plastic deformations take place (plastic system).

Analysis of Elastic System (Active condition)

The equations of motion of the retaining wall using d'Alembert's principle can be written in general terms as follows:

$$M\ddot{x} + \sum_{n}^{i=1} k_i \left[x + \{(H-\bar{h})-(i-1)\,\Delta h\}\,\theta \right] = F_o \operatorname{Sin} wt \qquad (1)$$

$$J\ddot{\theta} + \sum_{n}^{i=1} k_i \left[x + \{(H-\bar{h})-(i-1)\,\Delta h\}\,\theta \right] \{(H-\bar{h})-(i-1)\,\Delta h\} = 0 \qquad (2)$$

Put

$$\frac{\sum_{n}^{i=1} k_i}{M} = a \qquad (3)$$

$$\frac{F_0}{M} = a_0 \tag{4}$$

$$\frac{-\sum_{i=1}^{n} k_i \{(H-\bar{h}) - (i-1)\Delta h\}}{M} = b \tag{5}$$

and
$$\frac{\sum_{i=1}^{n} k_i \{(H-\bar{h}) - (i-1)\Delta h\}^2}{J} = c \tag{6}$$

The equations of motion of the rigid retaining wall can thus be written as;

$$\ddot{x} + ax = b\theta + a_0 \text{ Sin wt} \tag{7}$$

$$\ddot{\theta} + C\theta = \left(\frac{b}{r^2}\right) x \tag{8}$$

where $J = Mr^2$ and 'b' can be called as coupling coefficient because if $b=O$, the two equations become independent of each other.

For the coupled equations obtained, assume solution as

$$x = X \text{ Sin wt and } \ddot{x} = -w^2 X \text{ Sin wt}$$
$$\theta = \beta \text{ Sin wt and } \ddot{\theta} = -w^2 \beta \text{ Sin wt}$$

Substituting in equations of motion, we get

$$(-w^2+a)\, X = b\beta + a_0$$
$$(-w^2+c)\, \beta = \left(\frac{b}{r^2}\right) X$$

Solving these, we have

$$X = \frac{a_0}{(a-w^2) - \dfrac{b^2}{r^2\,(c-w^2)}} \tag{9}$$

$$\beta = \frac{a_0}{(a-w^2)\,(c-w^2)\,\dfrac{r^2}{b} - b} \tag{10}$$

Hence the solution becomes

$$x = \frac{a_0}{(a-w^2) - \dfrac{b^2}{r^2\,(c-w^2)}} \text{ Sin wt} \tag{11}$$

$$\theta = \frac{a_0}{(a-w^2)\,(c-w^2)\,\dfrac{r^2}{b} - b} \text{ Sin wt} \tag{12}$$

Therefore, the displacement of the top of rigid retaining wall is given by

$$X_{top} = x + (H-\bar{h})\,\theta$$

$$\therefore \qquad X_{top} = \left\{ \frac{r^2 (c-w^2) + b (H-\bar{h})}{(a-w^2)(c-w^2) \;\; r^2 - b^2} \right\} a_0 \text{ Sin } wt \tag{13}$$

Natural Frequencies:—Under free vibration condition, the equations of motion are;

$$\ddot{x} + ax = b\theta$$

$$\ddot{\theta} + c\theta = \left(\frac{b}{r^2}\right) x$$

Substituting the solution

$$x = X \text{ Sin } wt \text{ and } \ddot{x} = - w^2 X \text{ Sin } wt$$

$$\theta = \beta \text{ Sin } wt \text{ and } \ddot{\theta} = - w^2 \beta \text{ Sin } wt$$

Equations become

$$(-w^2+a)\, X = b\beta$$

$$(-w^2+c)\beta = \left(\frac{b}{r^2}\right) X$$

From these we get,

$$\frac{X}{\beta} = \frac{b}{a-w^2} \quad \text{and}$$

$$\frac{X}{\beta} = \frac{c-w^2}{\left(\frac{b}{r^2}\right)}$$

Equating

$$\frac{b}{a-w^2} = \frac{c-w^2}{\left(\frac{b}{r^2}\right)}$$

$$w^4 - (a+c)\, w^2 + \left(ac - \frac{b^2}{r^2} \right) = 0$$

Solving we get

$$w^2_{n1,2} = \tfrac{1}{2}(a+c) \pm \sqrt{\left(\frac{c \quad a}{2} \right)^2 + \left(\frac{b}{r}\right)^2} \tag{14}$$

Passive condition:—The ratio of stiffnesses on the compression and tension sides is denoted by η. Hence in the passive condition, the values of a, b and c get chan ged and these can be givan by

$$a = \eta\, (a)_a$$

$$b = \eta\, (b)_a$$

$$c = \eta\, (c)_a$$

The solution for this condition is similar to active condition described above.

Analysis of a Plastic System (Active Condition)

Assume that z_y and θ_y are the yield displacements occuring simultaneously in all springs, the equations of motion can be written as;

$$\ddot{x} + a\, z_y = b\, \theta_y + a_0 \operatorname{Sin} wt \tag{15}$$

$$\ddot{\theta} + c\theta_y = \left(\frac{b}{r^2}\right) z_y \tag{16}$$

Integrating the above equations twice, we get

$$x = (b\theta_y - a z_y)\, \frac{t^2}{2} - \frac{a_0 \operatorname{Sin} wt}{w^2} + C_1 t + C_2 \tag{17}$$

$$\theta = \left(\frac{b}{r^2}\, z_y - c\theta_y\right)\frac{t^2}{2} + c_3 t + c_4 \tag{18}$$

Let 't_e' be the time after which displacement of top of wall(y_{top}) is greater than yield displacement (y_d), then plastic system starts. Let x_e, $\dot{x}_e$, θ_e and $\dot{\theta}_e$ be the values corresponding to time, t_e and these be calculated by using the equations developed for elastic system. The following boundary conditions can be applied to evaluate the constants of integration;

i) $t = t_e, \quad \dot{x} = \dot{x}_e$

ii) $t = t_e, \quad x = x_e$

iii) $t = t_e, \quad \dot{\theta} = \dot{\theta}_e$

iv) $t = t_e, \quad \theta = \theta_e$

Therefore, we have

$$z_y = x_e$$

$$\theta_y = \theta_e$$

$$C_1 = \dot{x}_e - (b\theta_y - a\, z_y)\, t_e + \frac{a_0 \operatorname{Cos} wt_e}{w}$$

$$C_2 = \dot{x}_e - x_e t^2 + (b\theta_y - a z_y)\, \frac{t^2_e}{2} + \frac{a_0 \operatorname{Sin} wt_c}{w^2} - \frac{t_c a_0 \operatorname{Cos} wt_c}{w}$$

$$C_3 = \dot{\theta}_c - \left(\frac{b}{r^2}\, z_y - C\theta_y\right) t_c$$

$$C_4 = \theta_e + \left(\frac{b}{r^2}\, z_y - C\theta_y\right)\frac{t^2_c}{2} - \dot{\theta}_e\, t_e$$

Displacement of the top of rigid retaining wall is given by

$$x_{top} = x + (H - \bar{h})\, \theta$$

Passive Condition : The ratio of stiffnesses on the compression and tension sides is denoted by η. Hence in the passive condition, the value of a, b and c change and these can be given by,

$$a = \eta \, (a)_a$$
$$b = \eta \, (b)_a$$
$$c = \eta \, (c)_a$$

The solution is similar to the above procedure for active condition except the values of z_y and θ_y. In compression side (passive condition), the displacement for achieving yield condition are very large, hence in most of the cases plastic system for passive case is not considered.

APPLICATION OF METHODOLOGY

Displacements Under Dynamic Condition

In the present study a model wall shown in Fig. 2 (a) retaining medium dense sand has been choosen to predict its displacements under given dynamic condition by the proposed analysis;

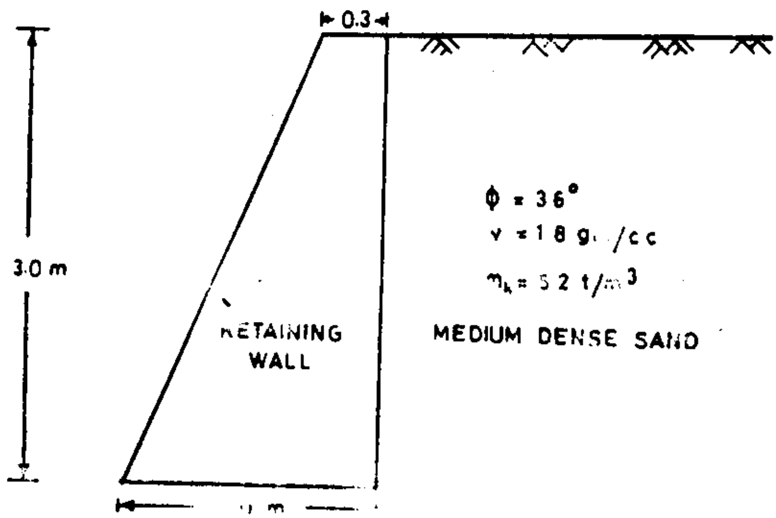

Fig. 2—(a) Retaining Wall Analysed

Data

Retaining wall :	H	=	3.0 m
	b'	=	1.0 m
	a'	=	0.3 m
	medium dense sand		
Backfill soil :	ϕ	=	36°
	γ	=	1.8 gm/cc
	n_h	=	52 t/m³ (active)
Yield displacement:	y_d	=	0.6 cm
Ground acceleration:	a_0	=	0.25 g
Time period :	T_p	=	0.3 sec.

Mathematical Model

Let the height of wall, H be divided into four equal number of segments ($\triangle h$ equal to 0.75 m). Let the backfill soil be idealized as springs fixed to the back of wall and to the fixed support. In this solution, five springs are considered. The mass of retaining wall is assumed to be lumped at centre of gravity. The c.g. is at a distance of $\bar{h}$ equal to $\frac{H}{3}\left(\frac{a'+2b'}{a'+b'}\right) = 1.23$ m above the base. The mathematical model is shown in Fig. 2 (b).

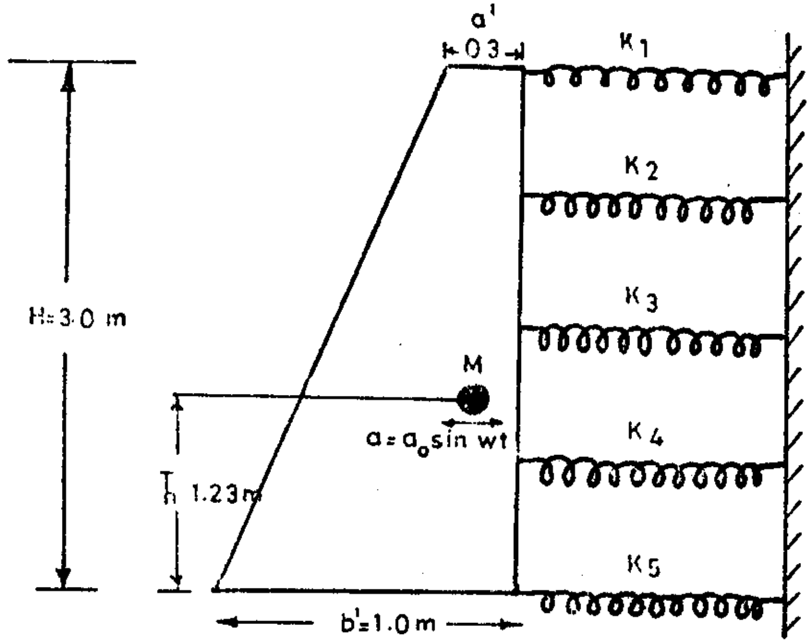

Fig. 2—(b) Mathematical Model

Spring Constants

Considering the backfill characteristics, the spring constants in active and passive cases can be found out and are given below;

Spring	Spring location w.r. to c.g. (m)	Spring constant active case (t/m)	Spring constant passive case (t/m)
K_1	1.77	4.88	9.75
K_2	1.02	29.25	58.50
K_3	0.27	58.50	117.00
K_4	—0.48	87.75	175.50
K_5	—1.23	53.63	107.25

Equations of Motion

The quantities required in the analysis are calculated as shown below;

$$M = \left(\frac{a'+b'}{2}\right) \times H \times \frac{2.306}{9.8} = 0.459 \text{ t/sec}^2\text{/m}$$

$$\bar{h} = \left(\frac{b'+2a'}{b'+a'}\right) \times \frac{H}{3} = 1.23 \text{ m}$$

$$r = 0.78 \text{ m}$$

$$J = Mr^2 = 0.279 \text{ (t—sec}^2\text{/m). m}^2$$

Using the above values, the values of a, b and c in active and passive cases can be determined;

Active Case	Passive Case;
$a = \frac{\Sigma k}{M} = 510.00$	$a = \frac{-\Sigma k}{M} = -1020.00$
$b = \frac{-\Sigma kh}{M} = 117.25$	$b = \left(\frac{-\Sigma kh}{M}\right) = -234.60$
$c = \frac{\Sigma kh^2}{J} = 542.35$	$c = \frac{-\Sigma kh^2}{J} = -1080.70$

The equations of motion can be written as

$$\ddot{x} + ax = b\theta + a_o \text{ Sinwt}$$

$$\ddot{\theta} + c\theta = \left(\frac{b}{r^2}\right) x$$

Natural Frequencies

The natural frequencies of the wall by considering the tension side are given by;

$$w^2_{n1,2} = \tfrac{1}{2}(a+c) \pm \sqrt{\left(\frac{c-a}{2}\right)^2 + \left(\frac{b}{r}\right)^2}$$

$$W_{n1} = 26.03 \text{ rad/sec}$$

$$w_{n2} = 19.36 \text{ rad/sec}$$

The natural time periods are therefore,

$$T_{n1} = 0.24 \text{ sec}$$

$$T_{n2} = 0.32 \text{ sec}$$

Dynamic Load

Earthquake motions are erratic and no two accelerograms are similar. The two main parameters of any ground motion are the amplitude of acceleration and the number of zero crossings in unit time. A very simple and convenient form of ground motion including the above two parameters is sinusoidal motion. Moreover, while proposing a method for analysing the liquefaction potential of sand deposits, Seed and Idriss (1967) contended that any given accelerogram can be considered equivalent to some definite number of cycles of loading of equal magnitude. Such idealizations have advantage thar after studying the effect of two parameters, the effect of a probable earthquake motion at any site, can be analysed. Because of the above advantages, sinusoidal ground motions are utilised in the present study.

$$\text{Given } a_0 = 0.25\ g = 2.45\ \text{m/sec}^2$$

$$T_p = 0.3\ \text{sec}, \quad w = \frac{2\pi}{0.3} = 20.94\ \text{rad/sec}$$

$$\therefore \quad a = 2.45\ \text{Sin}\ (20.94\ t)$$

Prediction of Displacements in Elastic System

The displacements in passive case ($t = 0$ to $t_{p/2}$) can be calculated as shown below;

$$x = X\ \text{Sin} wt$$
$$\theta = \beta\ \text{Sin} wt$$

$$\text{where } X = \frac{a_0}{(a - w^2) - \dfrac{b^2}{r^2(c - w^2)}} = -\ 1.751 \times 10^{-3}\ \text{m}$$

$$= -\frac{a_0}{(a - w^2)\ (c - w^2)\dfrac{r^2}{b} - b} = -\ 4.431 \times 10^{-4}\ \text{rad.}$$

Hence, we have

$$x = -\ 1.751 \times 10^{-3}\ \text{Sin}\ (20.94\ t)$$
$$\theta = -\ 4.431 \times 10^{-4}\ \text{Sin}\ (20.94\ t)$$

Time t (sec)	Translational Displacement x (mm)	Rotation (rad)	Disp. of wall at top due to rotation x_θ (mm)	Total disp. at top x_{top} (mm)
0	0	0	0	0
0.0375	−1.24	-3.1328×10^{-4}	−0.55	−1.79
0.0750	−1.75	-4.431×10^{-4}	−0.78	−2.53
0.1125	−1.24	-3.1328×10^{-4}	−0.55	−1.79
0.15	0	0	0	0

The displacements in active case $\left(t = \frac{t_p}{2} \text{ to } t_p\right)$ can be calculated as

$$x = X\ \text{Sin} wt$$
$$\theta = \beta\ \text{Sin} wt$$

$$\text{where } X = \frac{a_0}{(a - w^2) - \dfrac{b^2}{r^2\ (c - w^2)}} = -\ 1.6776 \times 10^{-3}\ \text{m}$$

$$\beta = \frac{a_0}{(a-w^2)\,(c-w^2)\,\frac{r^2}{b} - b} = 1.7871\times10^{-4} \text{ rad.}$$

Hence we have,

$$x = -1.6776\times10^{-2}\ \text{Sin}\ (20.94\ t)$$
$$\theta = -1.7871\times10^{-4}\ \text{Sin}\ (20.94\ t)$$

Time t (sec)	Translation displacement x (mm)	Rotation (rad)	Disp. of wall at top due to rotation $x\theta$ (mm)	Total disp. at top x_{top} (mm)
0.15	0	0	0	0
0.1875	11 86	1 2636 $\times10^{-4}$	0.22	12.08
0.2250	16.78	1.7871 $\times10^{-4}$	0.32	17.10
0.2625	11 86	1.2636 $\times10^{-4}$	2.22	12.08
0.3	0	0	0	0

The dynamic response of the retaining wall under elastic system is shown in Fig. 3 which indicates that the slip (permanent displacement)

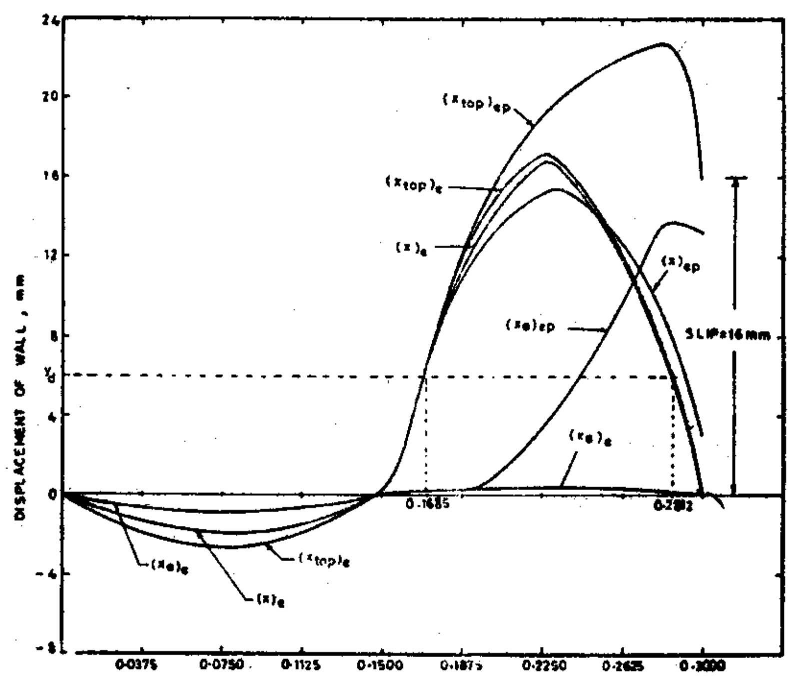

Fig. 3—Dynamic Response of Retaining Wall

after one cycle of ground motion is zero. It can be concluded that in elastic system after any number of cycles, the residual displacement would be zero.

Check for Plastic Conditions

When the displacement of wall (x_{top}) is greater than yield displacement (y_d), the valid system would be plastic. Therefore the equations of plastic system should be used. In passive case, yield displacement (y_d) is so large that plastic conditions do not arise and elastic system is considered In active case, to identify the time 't_e' after which plastic conditions exist, a line has been drawn for y_d value. From figure 4.3,

$t_e = 0.1688$ sec

$x_e = -1.677\times10^{-2}$ Sin $(20.94\ t_e) = 6{\cdot}4258\times10^{-3}$ m.

$\theta_e = 1.7871\times10^{-4}$ Sin $(20.94\ t_e) = 6.8452\times10^{-5}$ rad.

Based on the analysis

$Z_y = X_e = 6.4258\times10^{-3}$ m

$\theta_y = \theta_e = 6.8452\times10^{-5}$ rad.

$\dot{x}_e = 0.325$ m/sec

$\dot{\theta}_e = 3.4568\times10^{-3}$ red/sec

$C_1 = 0.7683$

$C_2 = -0.08$

$C_3 = -0.2$

$C_4 = 0.0166$

Hence the governing equations for displacement become,

$x = -1.6346t^2 - 5.587x10^{-3}$ Sin $(20.94t) + 0.7683t - 0.08$

$\theta = 0\ 6t^2 - 0.2t + 0.0166$

Time t (sec)	Translational displacement x (mm)	Rotation θ (rad)	Disp. of wall at top due to rotation x_θ, (mm)	Total disp. of wall at top x_{top} (mm)
0.1688	5.25	$6.3946x10^{-5}$	0.11	5.36
0.1875	10.54	$1.9375x10^{-4}$	0 34	10.88
0.2250	15.70	$1.975x10^{-3}$	3.50	19.20
0.2625	12.30	$5.443x10^{-3}$	9.63	21.93
0.2812	8.94	$7.804x10^{-3}$	13.81	22.75

From t = 0.2812 sec to t = 0.3 sec, the displacements are computed using the expressions obtained by solving the equations of motion under elastic condition taking boundary conditions satisfying the previously computed values at t = 0.2812 sec. It is found that after one cycle the displacement is equal to 16 mm and it can be called as slip. The total displacement after n cycles is equal to equal to n times the slip. The final translational displacement and rotation of the retaining wall are therefore known.

Comparison

Slip by the present analysis = 1.6 cm

If the number of cycles are 15, then

total displacement = 1.6x15 = 24 cm

According to solution given by

Nandakumaran (1973), total displacement = 21.30 cm

Therefore it can be concluded that the analysis proposed by Nandakumaran (1973) which is based on single degree freedom system underestimates the displacement.

CONCLUSIONS

The proposed method of analysis for predicting displacement of the wall under dynamic condition considers translationaland rotational motions simultaneously and therefore is more realistic. The method based on single degree freedom system underestimates the displacement.

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