Bulletin of the Indian Society of Earthquake Technology. Vol. 6, No. 1, pp 7–15 March, 1969

IMPACT DAMPERS

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Synopsis

The application of impact dampers for systems subjected to earthquake type excitations has been investigated. The parent system was idealised as a linear single degree of freedom system. Impact is produced by the collision of a single particle mass against the wall of a container rigidly mounted on the parent system. This study indicates that forces on parent system could be reduced by such devices.

Introduction

The use of impact dampers has so far been studied for systems subjected to steady state excitations(1,2,3)†. An impact damper primarily reorganises the vibrating pattern of a physical system. If the impact is so adjusted that the amplitude of vibration decreases, then the force on parent system is reduced.

An impact damper is a device which reduces the vibration amplitude of a system through the mechanism of momentum transfer by collision and conversion of mechanical energy into heat. It essentially consists of a mass particle within a container, which is fixed to the parent system, such that the particle has specified freedom to move relative to the container. The energy of the mass particle is dissipated in impact.

Here, a study has been made of the application of single mass impact dampers to linear single degree of freedom systems subjected to earthquake excitations. The equations of motion have been given and results obtained for various values of parameters involved in the problem. The efficiency of the impact damper has been worked out for various cases and among them, the maximum reduction was of the order of forty percent.

Equations of Motion

The parent system has been idealised as a linear single degree of freedom system having a mass M, viscous damping constant C and spring constant K. The single particle has a mass m which moves in a frictionless container and has a clearance d in which it is free to oscillate. A single translational component of the ground motion is only considered (Fig. 1).

Between impacts, the equation of motion of the parent system is given by,

$$M\ddot{X}_1 + C(\dot{X}_1 - \dot{y}) + K(X_1 - y) = 0 \qquad (1)$$

where X_1 is the absolute displacement of mass M and y is the ground displacement.

Dividing equation 1 by M and subtracting $\ddot{y}$ from both sides,

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† Refers to serial number of references listed at the end.

$$\ddot{Z}_1 = -(\ddot{y} + 2\,p\,\zeta\,\dot{Z}_1 + p^2 Z_1) \tag{2}$$

where $Z_1 = X_1 - y$, the displacement of the mass M, relative to the base.

$p = \sqrt{K/M}$, the undamped circular natural frequancy of the parent system, also, equal to $2\pi/T$, where T is the undamped natural period of the system, also, equal to $\sqrt{g/\delta_{st}}$ where δ_{st} is the static deflection of the parent system and g is the acceleration due to gravity.

$\zeta = \dfrac{C}{2\sqrt{KM}}$, percentage of critical damping of the parent system.

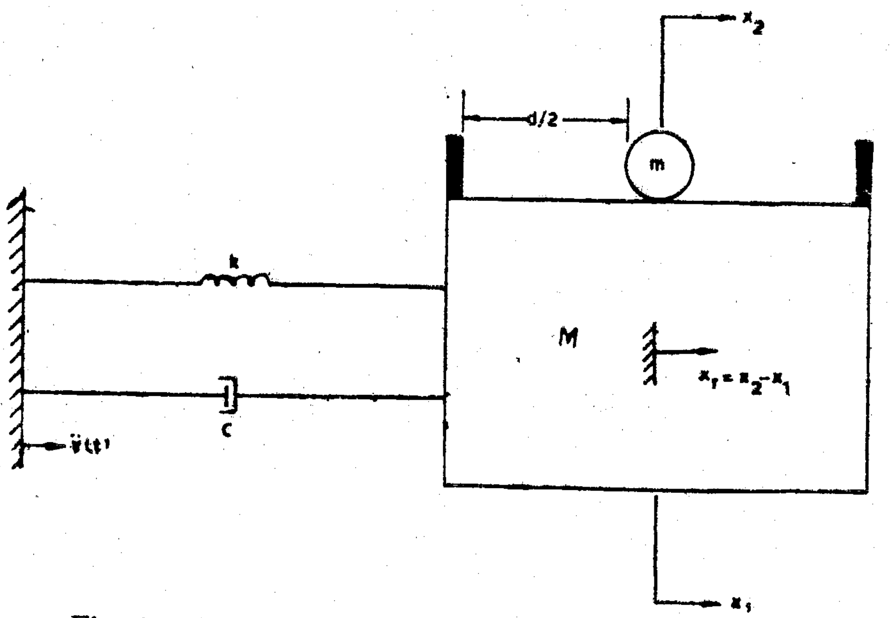

Fig. 1. A mathematical model of a damped elastic system with an impact damper

If X_2 is the absolute displacement of the particle m, then, between impacts, its equation of motion is given by $m\ddot{X}_2 = 0$ (as there is no friction and no spring force). Since m is not zero,

$$\ddot{X}_2 = 0 \tag{3}$$

If X_r is the relative displacement between the masses m and M then, $X_r = X_2 - X_1$

$$\text{and} \quad \ddot{X}_r = \ddot{X}_2 - \ddot{X}_1 = -\ddot{X}_1 \ (\text{as } \ddot{X}_2 = 0) \tag{4}$$

From equations 1 and 4,

$$\ddot{X}_r = 2\,p\,\zeta\,\dot{Z}_1 + p^2 Z_1 \tag{5}$$

Since the duration of the impact is very small compared to the period of vibration of the system, it has been assumed that during impact no change in displacement takes place and only velocity changes. If subscripts – and + indicate respectively quantities preceding and following an occurrence then,

$$\text{at} \quad t = t_{1-}\,;\ X_r = d/2,\ Z_1 = Z_{11-},\ \dot{Z}_1 = \dot{Z}_{11-}$$

and,

$$\text{at} \quad t = t_{1+}\,;\ X_r = d/2,\ Z_1 = Z_{11-},\ \dot{Z}_1 = \dot{Z}_{11+} \tag{6}$$

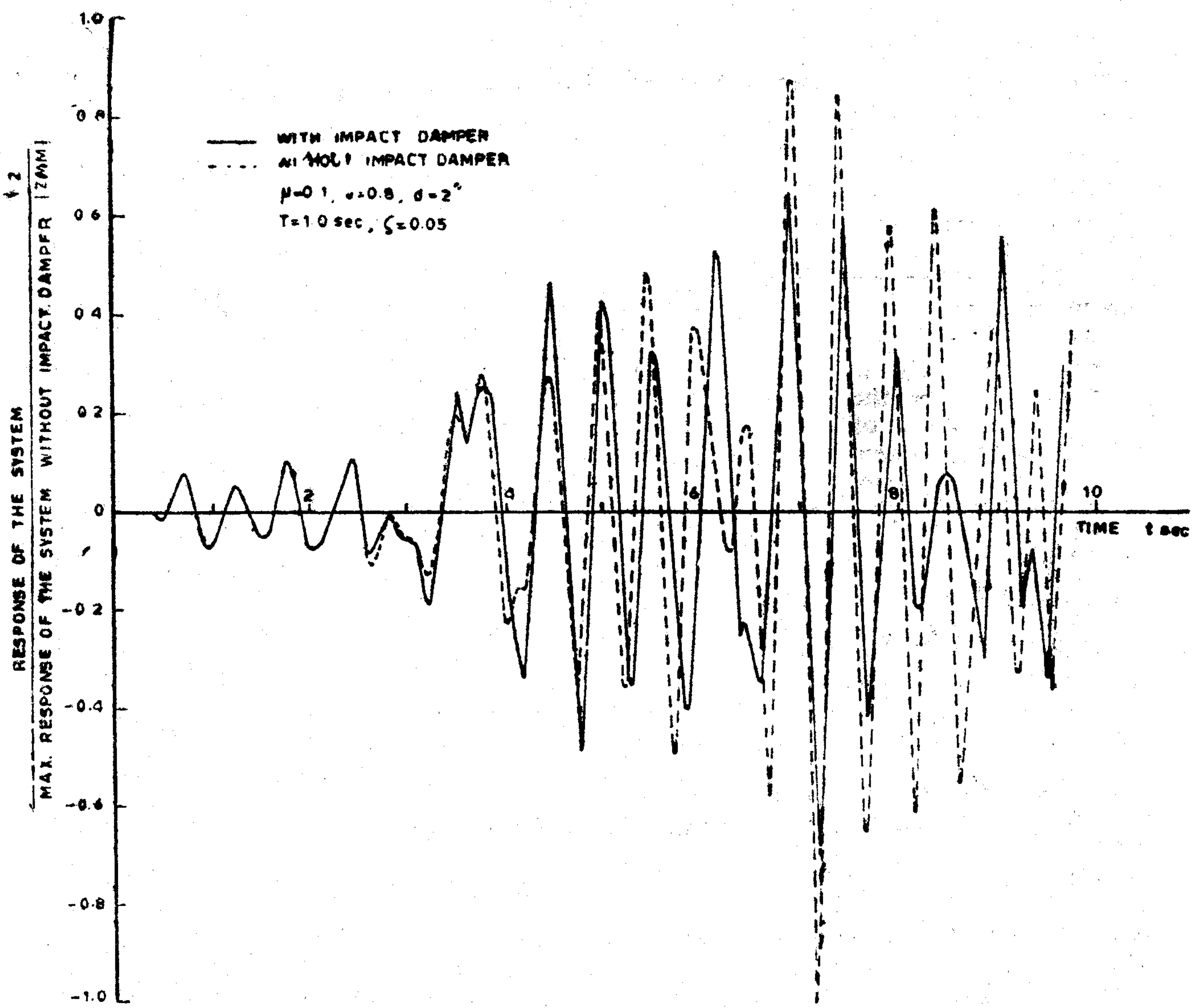

Fig. 2. Influence of impact damper, Taft earthquake.

During impact, momentum equation has to be satisfied. That is,

$$M \dot{X}_{1-} + m \dot{X}_{2-} = M \dot{X}_{1+} + m \dot{X}_{2+} \tag{7}$$

Subtracting $(M + m) \dot{y}$ from both sides of equation 7,

$$M (\dot{X}_{1-} - \dot{y}) + m (\dot{X}_{2-} - \dot{y}) = M (\dot{X}_{1+} - \dot{y}) + m (\dot{X}_{2+} - \dot{y}) \tag{8}$$

or $$M \dot{Z}_{1-} + m \dot{Z}_{2-} = M \dot{Z}_{1+} + m \dot{Z}_{2+}$$

From the definition of co-efficient of restitution, e,

$$\dot{X}_{r+} = - e \dot{X}_{r-} \tag{9}$$

Since $X_r = X_2 - X_1$ and $Z_1 = X_1 - y$, equation 9 can be represented as

$$\dot{Z}_{2+} - \dot{Z}_{1+} = -e\,(\dot{Z}_{2-} - \dot{Z}_{1-}) \tag{10}$$

From equations 8 and 10,

$$\dot{Z}_{1+} = \dot{Z}_{1-} + \frac{\mu\,(1 + e)}{1 + \mu}\,\dot{X}_{r-} \tag{11}$$

where μ = mass ratio, m/M.

Summarising, equations 2 and 5 govern the motion of the masses between two successive impacts. These equations could be solved, using numerical techniques, to obtain velocities $\dot{Z}_1$ and $\dot{X}_r$ and displacements Z_1 and X_r at any instant of time. At the time of impact, defined by $X_r = d/2$, displacements, Z_1 and X_r do not change but velocities, which become discontinuous functions, do change and are given by equations 9 and 11.

Variables

The following variables are involved in the problem :—(i) Undamped natural period, T, and percentage of critical damping, ζ, of the parent system, (ii) ratio of mass of particle to that of parent system, μ, (iii) co-efficient of restitution, e, (iv) clear distance, d, of the container in which the particle is free to oscillate smoothly and (v) ground motion, $\ddot{y}$.

T has two values, namely 0.5 and 1.0 second and ζ was taken as 0.05. μ has values ranging between 0.1 and 0.3 and e between 0.0 to 1.0. d/δ_{st} had values between 0.30 and 2.00. Two ground motion data, namely, (i) NS component of El Centro, May 18, 1940, and (ii) S 21 W component of Taft, July 21, 1952, were utilised.

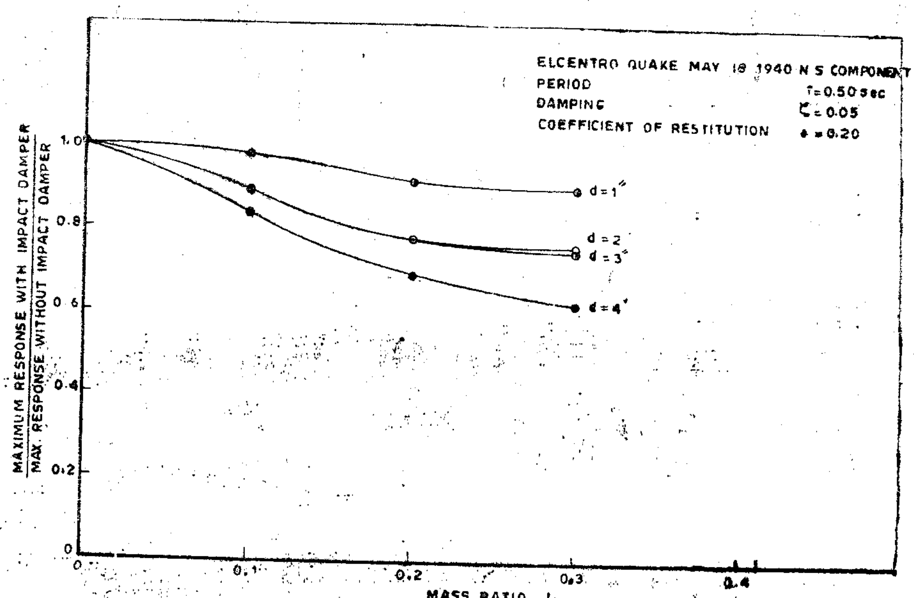

Fig. 3. Effect of mass ratio on the response of the system

The lateral shear force on the parent system is directly proportional to its displacement Z_1, relative to the base. The ratio of displacement response of the parent system with impact damper to that without it has been worked out for all the cases. If Z_M denotes maximum displacement with impact damper and Z_{MM} without it, then the above ratio is equal to Z_M / Z_{MM} and the efficiency of the impact damper is given by $1-Z_M / Z_{MM}$.

It is known that for a linear system, response decreases with increase in damping. Therefore, the influence of the impact damper can also be represented as an increase in the damping of the system. The results are also expressed as an increase in percentage of critical damping over that already inherent in the system (namely, above 0.05).

Results

Tables 1, 2, 3 and 4 give the efficiency of impact damper as well as equivalent increase in damping factor for the various cases that have been solved. The influence of the various parameters are discussed below.

(a) *Time wise Response*

Figure 2 shows a typical time-wise response of the parent system with and without impact damper for a particular combination of variables. It is seen that the impact damper reduces the amplitude of vibration and is particularly effective at large amplitudes.

(b) *Effect of Mass Ratio*

Figures 3 and 4 show the influence of mass ratio on the response of the system. It is seen that the efficiency of damper increases with increase in mass ratio.

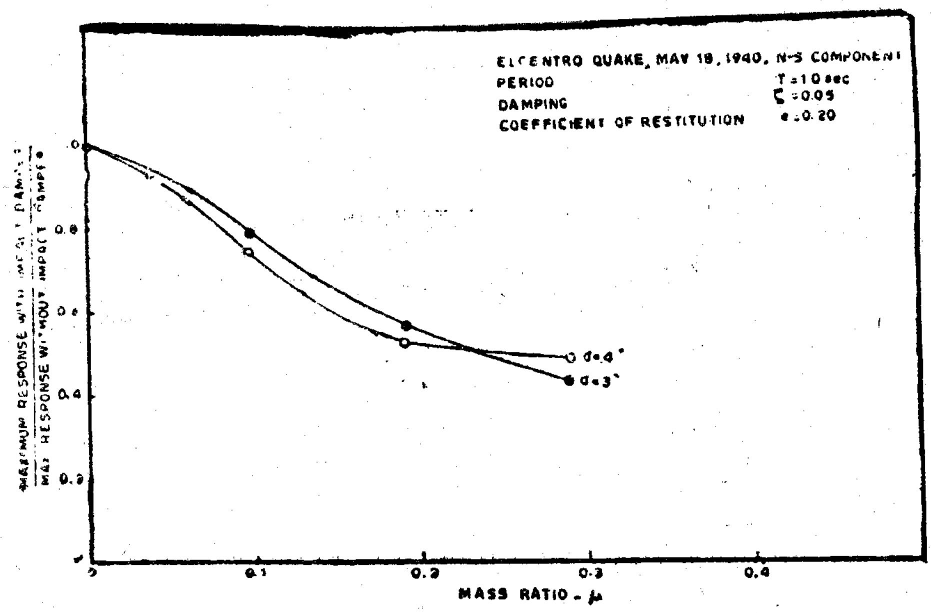

Fig. 4. Effect of mass ratio on the response of the system

(c) *Effect of Co-efficient of Restitution*

The variation of response of primary system with coefficient of restitution is given in figures 5 and 6. No definite pattern of variation is perceptible from these results.

(d) *Effect of Clearance*

From the tables, it could be seen that for the various cases tried, the efficiency of damper generally increases with clearance.

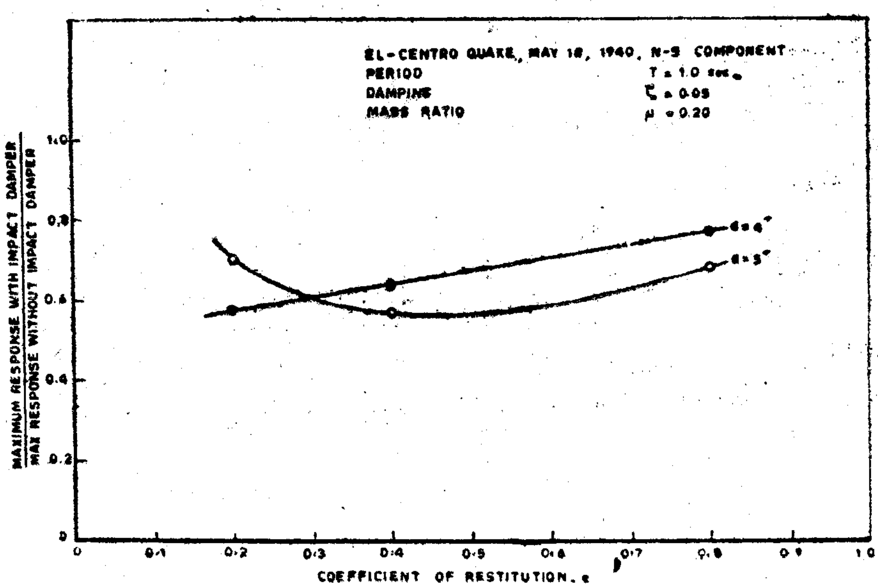

Fig. 5. Effect of coefficient of restitution on the response of the system

(e) *Effect of Period*

Only two periods have been tried. It is seen that impact damper is more efficient for longer period systems.

Conclusion

The analysis indicates that the impact damper could be used to reduce the response of systems subjected to earthquake type excitations. Among the various cases tried, the maximum reduction in response was of the order of forty percent. Probably, a multiple particle impact damper with variable clearances may give greater reduction in response.

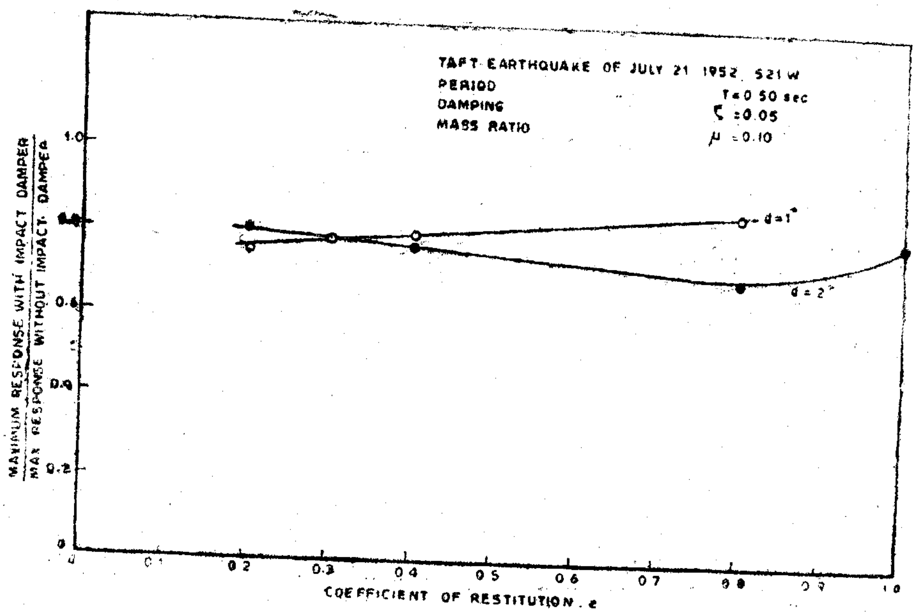

Fig. 6. Effect of coefficient of restitution on the response of the system

Acknowledgement

The authors are thankful to the Director, School of Research and Training in Earthquake Engineering, University of Roorkee, Roorkee for the facilities provided during this investigation.

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TABLE 1

Maximum Displacement Response of the System to El Centro, Earthquake, May 18, 1940, N–S Component.

Period of primary system $T = 0.5$ sec.

Damping of primary system $\zeta = 0.05$

Maximum displacement response of primary system alone $Z_{MM} = 2.345$ inch

Static deflection $\delta_{st} = 2.450$ inch

Case No.	Mass Ratio μ	Coeff. of Restitution e	Clearance d (inch)	d/δ_{st}	Max. Disp. Z_M (inch)	Efficiency $1-\frac{Z_M}{Z_{MM}}$	Equivalent increase in Damping
1	0.10	0.2	1.0	0.408	2.464	2.70	0.665
2	0.10	0.3	1.0	0.408	2.478	2.20	0.543
3	0.10	0.4	1.0	0.408	2.482	2.00	0.493
4	0.10	0.2	2.0	0.816	2.248	11.25	2.775
5	0.10	0.3	2.0	0.816	2.245	11.34	2.800
6	0.10	0.4	2.0	0.816	2.232	11.86	2.920
7	0.10	0.2	3.0	1.225	2.238	11.62	2.870
8	0.10	0.3	3.0	1.225	2.244	11.41	2.810
9	0.10	0.4	3.0	1.225	2.254	11.00	2.710
10	0.10	0.2	4.0	1.634	2.110	16.70	4.120
11	0.10	0.3	4.0	1.634	2.170	14.30	3.520
12	0.10	0.4	4.0	1.634	2.225	12.13	3.000
13	0 20	0.2	1.0	0.408	2.317	8.52	2.100
14	0.20	0.3	1.0	0.408	2.292	9.50	2 340
15	0.20	0.4	1.0	0.408	2.335	7.82	1.930
16	0.20	0.2	2.0	0.816	1.955	22.81	5.750
17	0.20	0.3	2.0	0.816	1.969	22.27	5.610
18	0.20	0.4	2.0	0.816	1.956	22.78	5.765
19	0.20	0.2	3.0	1.225	1.966	22.38	5.625
20	0.20	0.3	3.0	1.225	1.989	21 47	5.361
21	0.20	0.4	3.0	1.225	2.005	20.82	5.160
22	0.20	0.2	4.0	1.634	1.746	31.06	8.320
23	0.20	0.3	4.0	1.634	1.775	29.90	7.960
24	0.20	0.4	4.0	1.634	1.828	27.80	7.310

Table 2

Maximum Displacement Response of the System to El Centro, Earthquake May 18, 1940, N–S Component.

Period of primary system, $T = 1.0$ sec.

Damping of primary system, $\zeta = 0.05$

Maximum displacement response of primary system alone, $Z_{MM} = 4.591$ inch

Static Deflection, $\delta_{st} = 9.775$ inch

Case No.	Mass Ratio μ	Coeff. of Restitution e	Clearance d (inch)	d/δ_{st}	Maxm. Disp. Z_M (inch)	Efficiency $1-\frac{Z_M}{Z_{MM}}$	Equivalent increase in damping
1	0.1	0.2	3.0	0.306	3.699	19.41	2.900
2	0.1	0.4	3.0	0.306	3.920	14.60	2.185
3	0.1	0.2	4.0	0.410	3.484	24.10	3.600
4	0.1	0.4	4.0	0.410	3.724	18.88	2.825
5	0.1	0.8	4.0	0.410	4.252	7.37	1.103
6	0.2	0.2	3.0	0.306	2.852	37.87	6.367
7	0.2	0.4	3.0	0.306	3.189	30.52	4.570
8	0.2	0.2	4.0	0.410	2.657	42.12	7.970
9	0.2	0.4	4.0	0.410	2.901	36.80	6.040
10	0.2	0.8	4.0	0.410	3.526	23.18	3.470
11	0.2	0.2	5.0	0.512	3.234	29.55	4.430
12	0.2	0.4	5.0	0.512	2.588	43.62	8.120
13	0.2	0.8	5.0	0.512	3.104	32.39	4.850

Table 3

Maximum Displacement Response of the System to Taft Earthquake, July 21, 1952 S21 W, Component.

Period of primary system, $T = 0.5$ sec.

Damping of primary system, $\zeta = 0.05$

Maximum displacement response of primary system alone, $Z_{MM} = 0.909$ inch

Static Deflection, $\delta_{st} = 2.450$ inch

Case No.	Mass Ratio μ	Coeff. of Restitution e	Clearance d (inch)	d/δ_{st}	Maxm. Disp. Z_M (inch)	Efficiency $1-\frac{Z_M}{Z_{MM}}$	Equivalent increase in damping
1	0.1	0.2	1.0	0.408	0.728	20.00	3.820
2	0.1	0.3	1.0	0.408	0.705	22.50	4.295
3	0.1	0.4	1.0	0.408	0.716	21.30	4.070
4	0.1	0.8	1.0	0.408	0.775	14.72	2.810
5	0.1	0.2	2.0	0.816	0.679	25.25	4.870
6	0.1	0.3	2.0	0.816	0.704	22.64	4.310
7	0.1	0.4	2.0	0.816	0.693	23.80	4.540
8	0.1	0.8	2.0	0.816	0.630	30.69	6.402
9	0.1	1.0	2.0	0.816	0.721	20.66	3.940
10	0.1	0.0	5.0	2.040	0.904	0.54	0.103
11	0.1	1.0	5.0	2.040	0.902	0.76	0.150
12	0.2	0.2	1.0	0.408	0.664	26.96	5.237
13	0.2	0.3	1.0	0.408	0.672	26.08	4.980
14	0.2	0.2	2.0	0.816	0.595	34.52	7.600
15	0.2	0.2	3.0	1.225	0.883	2.92	0.557
16	0.2	0.2	5:0	2.040	0.893	1.71	0.326

TABLE 4

Maximum Displacement Response of the System to Taft Earthquake, July 21, 1952, S21 W, Component.

Period of primary system	$T = 1.0$ sec.
Damping of primary system	$\zeta = 0.05$
Maximum displacement response of primary system alone	$Z_{MM} = 1.778$ inch
Static deflection	$\delta_{st} = 9.775$ inch

Case No.	Mass Ratio μ	Coeff. of Restitution e	Clearance d (inch)	d/δ_{st}	Max. Disp. Z_M (inch)	Efficiency $1-\frac{Z_M}{Z_{MM}}$	Equivalent increase in Damping
1	0.1	0.0	3.0	0.306	1.488	16.31	3.980
2	0.1	0.0	5.0	0.512	1.631	8.27	2.020
3	0.1	0.1	5.0	0.512	1.614	9.22	2.250
4	0.1	0.2	5.0	0.512	1.573	11.53	2.810
5	0.1	0.4	5.0	0.512	1.505	15.34	3.740
6	0.1	1.0	5.0	0.512	1.451	18.40	4.440
7	0.2	0.2	4.0	0.410	1.315	26.04	7.310
8	0.2	0.4	4.0	0.410	1.282	27.93	8.100
9	0.2	0.8	4.0	0.410	1.415	20.46	5.000
10	0.2	0.0	5.0	0.512	1.558	12.36	3.013
11	0.2	0.1	5.0	0.512	1.497	15.83	3.860
12	0.2	0.2	5.0	0.512	1.445	18.75	4.575
13	0.2	0.4	5.0	0.512	1.434	19.38	4.720
14	0.2	0.8	5.0	0.512	1.411	20.65	5.060
15	0.2	1.0	5.0	0.512	1.400	21.28	5.325