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SEISMIC ANALYSIS OF A NO-BREAK SET

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SYNOPSIS

In a major powerhouse, No Break Set is installed to ensure an uninterupted supply of power to vital equipment and systems so that in the event of failure of the normal power supply there is no break in the various operations. The function of such a set is only for the specific period that elapses between the failure of normal power supply and the taking over of the supply by the storage battery system. The function of this 'No-Break Set' is therefore vital and it is expected that such a set would remain in top condition all the time particularly during a seismic event since power is likely to go off in such an eventuality. This paper presents the results of seismic analysis carried out for a typical no break set proposed to be installed in an active seismic region.

INTRODUCTION

A 325 KVA No-Break system is proposed to be installed in a seismic region for which design earthquake parameters have been specified. The set consists of an Alternator, a DC motor and a Pony motor. Each one of these is fixed to a common mild steel bed-plate through bolts. The bed-plate consists of a grillage of beams of box section resting on a concrete foundation and is held in place by bolts grouted into the concrete. The stators of the Alternator-motor set are very stiff and act as a mass and transfer their seismic forces to the bed-plate as a rigid body.

A detailed vibration analysis was carried out to arrive at the seismic forces to be considered for the design of the no-break set to examine its suitability for the prescribed seismic environment. The details of analysis carried out for the set are described in the present paper.

MOTOR-ALTERNATOR SYSTEM

The stators of Alternator, DC motor and Pony motor are massive and rigid and are treated as rigid bodies subjected to base acceleration. This assumption is justified as even the rotors, which are considered to be relatively flexible as compared to the stators, themselves turn out to be quite rigid for the frequency range of interest of earthquake ground motion.

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Mathematical Model of the Rotor System

The rotor shaft is of circular cross section but the diameter varies along the length. It can be idealised as an assemblage of finite beam elements. The rotor is supported on bearings which are idealised as spring supports.

The number of beam elements used in the discretisation are 31 in Alternator part, 40 in DC motor part and 17 in Pony motor part. Fig. 1 shows the line diagram of the model and Table—I gives the member properties.

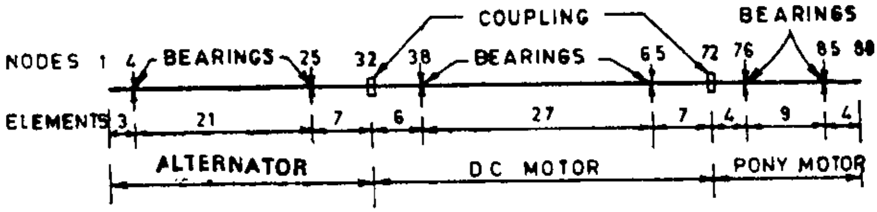

Fig. 1. The Alternator—DC Motor-Pony motor system showing node sequence.

Table—I Properties of the Beam Elements of the Rotor System

Elastic Modulus $(E) = 2 \times 10^6$ kg/cm^2

Poisson's ratio $(\nu) = 0.30$

Element	Length (cm)	Area of X-section (cm^2)	Moment of Inertia (cm^4)
1-2	5.4	38.48	117.86
2-3	1.6	60.82	294.37
3-4	2.25	70.68	399.82
4-5	2.25	70.88	399.82
5-6	5.00	95.07	718.69
6-7	5.00	102.07	829.07
7-8	5.00	102.07	829.07
8-9	1.00	113.10	1017.88
9-10	2.00	148.49	1754.61

Element	Length (cm)	Area of X-section (cm^2)	Moment of Inertia (cm^4)
10–11	3.00	258.02	5297.63
11–12	3.00	429.13	14654.68
12–13	10.00	530.93	22431.76
13–14	10.00	530.93	22431.76
14–15	10.00	530.93	22431.76
15–16	10.00	530.93	22431.76
16–17	10.00	530.93	22431.76
17–18	10.00	530.93	22431.76
18–19	3.00	363.05	10488.75
19–20	5.40	314.16	7853.98
20–21	5.40	314.16	7853.98
21–22	5.50	314.16	7853.98
22–23	5.80	181.46	2620.26
23–24	6.50	167.42	2230.39
24–25	2.75	113.10	1017.88
25–26	2.75	113.10	1017.88
26–27	1.90	105.00	888.88
27–28	3.00	98.52	772.40
28–29	5.25	95.03	718.69
29–30	5.25	95.03	718.69
30–31	5.25	95.03	718.69
31–32	5.25	95.03	718.69
32–33	5.25	95.03	718.69
33–34	5.25	95.03	718.69
34–35	5.25	95.03	718.69
35–36	5.25	95.03	718.69

Element	Length (cm)	Area of X-section (cm^2)	Moment of Inertia (cm^4)
36–37	2.60	98.52	772.40
37–38	1.90	105.68	888.88
38–39	4.75	113.10	1017.88
39–40	2.75	113.10	1017.88
40–41	4.90	141.03	1582.67
41–42	4.90	141.03	1582.67
42–43	5.00	176.71	2485.05
43–44	1.50	237.24	4478.88
44–45	1.50	258.16	5303.40
45–46	4.00	307.91	7544.50
46–47	4.00	307.91	7544.50
47–48	10.04	283.53	6397.12
48–49	10.04	283.53	6397.12
49–50	10.04	283.53	6397.12
50–51	10.04	283.53	6397.12
51–52	10.04	283.53	6397.12
52–53	4.00	283.53	6397.12
53–54	4.00	283.53	6397.12
54–55	5.00	243.28	4710.00
55–56	5.00	243.28	4710.00
56–57	5.00	237.79	4499.53
57–58	5.00	237.79	4499.53
58–59	5.00	237.79	4499.53
59–60	5.00	237.79	4499.53
60–61	5.00	237.79	4499.53
61–62	4.20	237.79	4499.53

Element	Length (cm)	Area of X-Section (cm^2)	Moment of Inertia (cm^4)
62–63	2.00	145.27	1679.29
63–64	4.50	141.03	1582.67
64–65	4.50	141.03	1582.67
65–66	4.50	141.03	1582.67
66–67	4.75	113.10	1017.88
67–68	1.90	105.68	888.80
68–69	2.60	95.52	772.40
69–70	4.00	44.18	155.32
70–71	5.00	44.18	155.32
71–72	5.00	44.18	155.32
72–73	7.00	44.18	155.32
73–74	7.00	44.18	155.32
74–75	3.59	50.13	200.00
75–76	4.17	50.26	201.10
76–77	6.00	69.39	283.20
77–78	5.70	75.42	452.70
78–79	14.80	63.61	322.10
79–80	14.80	63.61	322.10
80–81	5.00	62.21	307.90
81–82	6.00	60.82	294.30
82–83	4.00	56.74	256.20
83–84	4.30	54.10	232.90
84–85	3.76	38.48	117.80
85–86	3.50	38.37	117.10
86–87	2.74	14.12	15.86
87–88	4.00	1.96	0.30
88–89	4.00	1.96	0.30

The longitudinal axes (which is the axis of rotation) of all the elements coincide and hence local and global axes of all elements could be taken as the same. The number of degrees of freedom per node of a beam element in space is six. However, in quite a few situations, for the purpose of analysis the number of degree of freedom could be reduced without loss of accuracy. In this case, for comparative study, the degrees of freedom per node considered were two, three, four and six.

Due to symmetry, the results describing the dynamic characteristics, are seen to be identical for vibrations in the two transverse directions. Further, there is no torsional restraint if the couplings are assumed as a rigid mass point. Alternatively, if the coupling is assumed as a finite torsional spring in the axial direction and as rigid in the translational direction, the system has a long torsional period having uncoupled motion with respect to other degrees of freedom. Also, since earthquake motions are defined only for the three translational degrees of freedom, there are no stresses produced by this assumed torsional mode due to its non-excitation by earthquakes. Thus, the system can be basically analysed as one with three degrees of freedom per node.

The element stiffness matrix for a beam element with 3 degrees of freedom (longitudinal and transverse deformation and a rotation about a corresponding axis) per node is given below (Przemiencki 1968),

$$[K] = \begin{bmatrix} C_1 & 0 & 0 & -C_1 & 0 & 0 \\ & C_2 & C_3 & 0 & -C_2 & C_3 \\ & & C_4 & 0 & -C_3 & C_5 \\ & & & C_1 & 0 & 0 \\ & & & & C_2 & -C_3 \\ & & SYM & & & C_4 \end{bmatrix}$$

where

$$C_1 = \frac{EA}{L};$$

$$C_2 = \frac{12EI}{L^3(1+\alpha)};$$

$$C_3 = \frac{C_2 \cdot L}{2};$$

$$C_4 = \frac{(4+\alpha)EI}{L(1+\alpha)};$$

$$C_s = \frac{(2-\alpha)EI}{L(1+\alpha)};$$

$$\alpha = \frac{12EI}{GA_sL^2}$$

and

E — Elastic Modulus of the material of the element,

G — Modulus of Rigidity of the element material,

A — Area of cross section of the element,

A_s — Effective area for shear,

I — Moment of Inertia of the beam element,

L — Length of the element.

The generalised displacement and force vectors for the element are as follows:

$$\{\Delta\}^T = \{u_i, v_i, \ldots, \theta_j\}^T$$

and

$$\{F\}^T = \{F_{xi}, \ldots, F_{\theta j}\}^T$$

where

u = Axial displacement i.e. displacement along longitudinal axis.

v = Transverse displacement i.e. displacement along transverse axis.

θ = Rotation about an axis perpendicular to the transverse axis.

F_x = Axial force.

F_y = Shear force.

F_θ = Bending moment.

Since the shaft has been divided into a number of elements and only the first few modes of vibration are needed in dynamic analysis, the mass matrix may be obtained by lumping the generalized mass at nodes resulting in a diagonal matrix,

$$[M] = \begin{bmatrix} m_0 & & & & & \\ & m_0 & & & 0 & \\ & & I_{m0} & & & \\ & & & m_0 & & \\ & 0 & & & m_0 & \\ & & & & & I_{m0} \end{bmatrix}$$

where m_0 equals sum of half the mass, I_{m0} is sum of half the mass moment of inertia of the adjacent elements connected to the node.

The bearings have been represented by linear springs and the values adopted at various nodes are given in Table—II. (The range of values of spring constants were given by the manufacturers).

Table—II Stiffness Values Used for the Bearings of the Rotor System

Node No.	Type of Bearings	Stiffness kg/cm	
		Axial	Radial
4	Roller	10^{10}	1246000
25	Ball	33000	358000
39	Roller	10^{10}	1429600
66	Ball	33000	355200
76	Roller	10^{10}	1118600
85	Ball	37900	239500

Assembling the element stiffness and mass matrices and applying the boundary conditions, the equation of motion for free vibration is given by

$$[M]\{\ddot{x}\} + [K]\{x\} = \{o\} \tag{1}$$

The natural frequencies and mode shapes would be obtained by solving the eigen-value problem (Chandrasekaran, 1974) represented by,

$$[K]\{\phi\} = p^2 [M]\{\phi\} \tag{2}$$

in which p is the circular natural frequency and ϕ the corresponding mode shapes. The values of frequencies for the system are computed and given in Table—III for the first three modes of vibration. The corresponding mode shapes are shown in Fig. 2.

Earthquake Motion and Response

The earthquake motion considered for analysis is represented by an accelerogram which has a peak ground acceleration of 0.3 g in horizontal direction. For vertical direction, the motion is taken as 2/3 of that in horizontal direction. The waveform of the motion corresponded to that recorded at El Centro, U.S.A., NS component of May 18, 1940.

Table—III Natural Frequencies of Rotor System

Mode	Frequency Hz	Mode Participation Factor
1	78.23	1.095
2	105.60	1.084
3	170.26	1.076

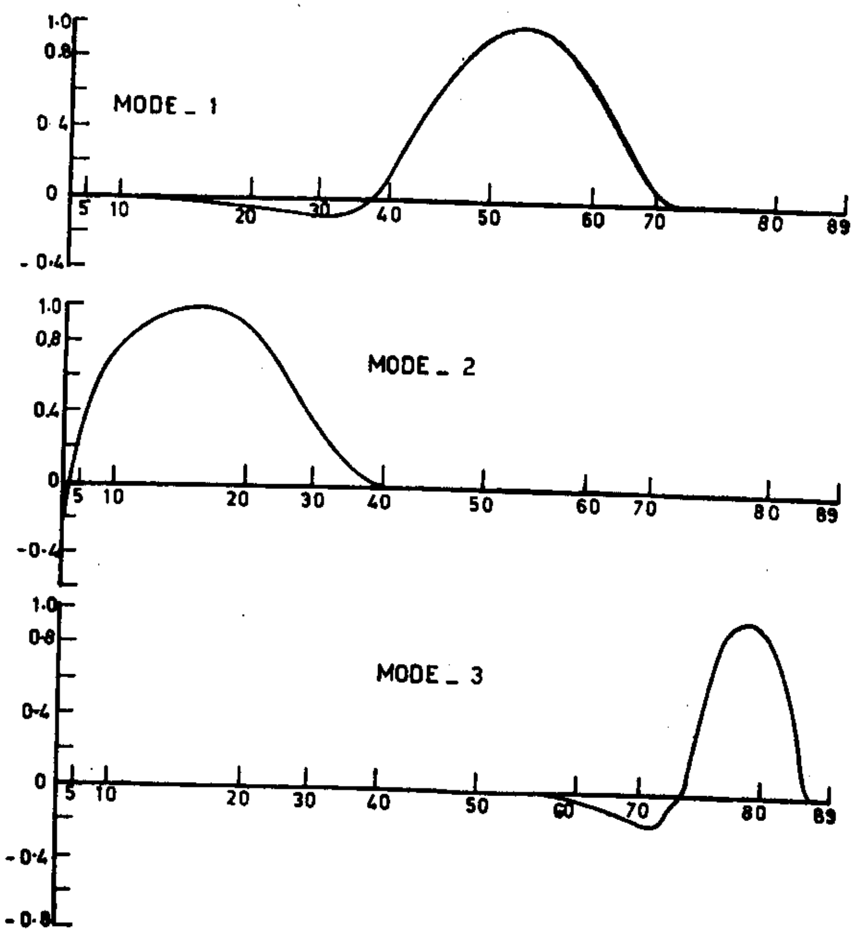

Fig. 2. First three mode shapes.

In an earthquake, although the ground moves simultaneously in all three directions, yet due to random nature of ground motion, the peaks in all the three components do not occur simultaneously. In this system, horizontal component parallel to longitudinal axis would mainly excite the axial mode which is quite rigid compared to the transverse modes (Also, the first few modes did not indicate a predominantly axial vibration).

The equation of motion of a system subjected to horizontal earthquake motion is given by

$$[M]\{\ddot{Z}\} + [C]\{\dot{Z}\} + [K](Z) = -[M]\{\alpha_h\}\cdot\ddot{y}_h \tag{3}$$

Where $\{Z\}$ is the relative displacement of nodes with respect to the base, $\ddot{y}_h$ is the time history of ground motion, and

$$\{\alpha_h\}^T = \{0, 1, 0, 0, 1, 0, \ldots\}^T$$

The solution of equation in each mode of vibration is given by

$$Z_i^{(r)} = \phi_i^{(r)}\cdot C_r\cdot S_d^{(r)} \tag{4}$$

in which $\phi_i^{(r)}$ = mode shape coefficient at node i in r^{th} mode of vibration.

C_r = mode participation factor

$$= \frac{\{\phi_i^{(r)}\}^T [M] \{\alpha_h\}}{\{\phi_i^{(r)}\}^T [M] \{\phi_i^{(r)}\}}$$

$S_d^{(r)}$ = spectral displacement in r^{th} mode of vibration corresponding to frequency p_r and damping ζ_r in that mode and for ground motion $\ddot{y}_h$

$$= \frac{1}{p_r\sqrt{1-\zeta^2}} \left| \int_0^t \ddot{y}(\tau) \exp\{-p_r\zeta_r(t-\tau)\} \times \right.$$

$$\left. \times \sin p_r\sqrt{1-\zeta^2}\,(t-\tau)d\tau \right|_{max}$$

The response for vertical motion can also be similarly computed and the total response obtained by timewise superposition.

RESULTS OF ANALYSIS

Referring to Table—III, it is seen that the fundamental frequency itself is quite high as compared to the frequencies of interest in earthquake records. Thus the rotor system can be treated as rigid and as far as seismic effects are concerned, there would be negligible dynamic magnification, since there is no possibility of even a quasi-resonance occurring in this condition.

The displacement response in first three modes of vibration has been obtained and using these values axial forces, shears and bending moments at various section, have been computed and plotted in Fig. 3. The maximum values of displacement and stresses in the first three modes are given in Table—IV, and are seen to be very small.

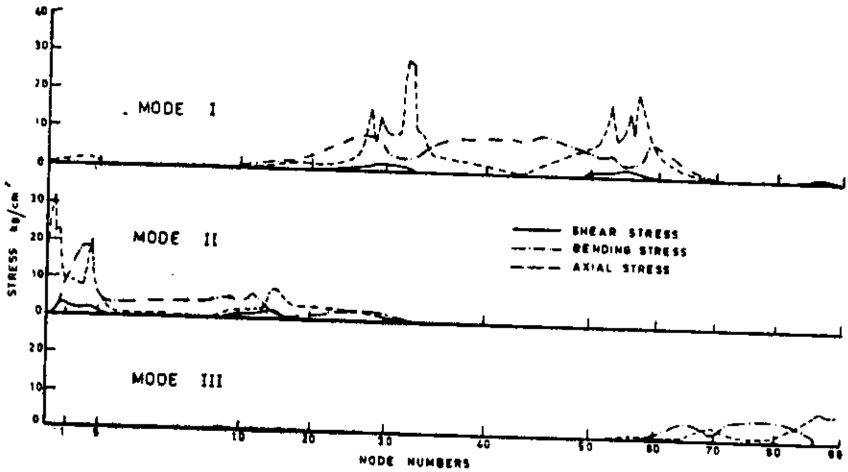

Fig. 3. Axial, Shear and Bending Stresses in first three modes of vibration.

Table—IV Maximum Response of the Rotor System in First Three Modes

Mode	Displacement (cm)	Bending Stress (kg/cm^2)	Shear Stress (kg/cm^2)	Axial Stress (kg/cm^2)
1	1.33×10^{-2}	10.27	2.06	29.06
2	7.25×10^{-3}	18.95	3.37	31.51
3	2.76×10^{-3}	5.11	0.68	8.38

CONCLUSION

It can be seen that the maximum displacements and stresses obtained are so small that they are of no consequence to the efficient performance of the rotor system during earthquakes.

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