

A NOTE ON THE FUNDAMENTAL PERIOD OF CONCRETE CHIMENEYS

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Using Rayleigh's minimum principle computation of fundamental period of concrete chimneys is suggested. Based on the results of the above principle, a prediction equation is proposed for reckoning the period rapidly in the design office. The equation yields reasonable values for the period and is simple to work with.

INTRODUCTION

Fundamental period is an important dynamic property of the chimney necessary for use in SEAOC code (Structural Engineers' Association of California), Rumman's and IS 18931975 code procedures for the design of chimneys subjected to earthquake effects. Various codes in the world recommend empirical equations for reckoning the same. 1,2,3 In this note a rational method is advanced based on Rayleigh's minimum principle for computing the period in a realistic manner. Finally a prediction equation is proposed for rapidly estimating the period in the design office.

RAYLEIGH'S MINIMUM PRINCIPLE

Rayleigh's principle is a powerful tool for finding the fundamental frequency of a vibrating system with satisfactory results. This principle leads to upper bound solution. For a system in which the mass is continuously distributed, the fundamental frequency can be found using the quotient

$$p^2 = \frac{Eg \int I_x \left(\frac{d^2 y}{dx^2} \right)^2 dx}{\int w_x y^2 dx} \dots\dots(1)$$

where

P = fundamental frequency in radians per sec.

y = shape function

I_x = moment of inertia function

W_x = self weight function

The period T is then given by the expression

$$T = \frac{2\pi}{p} \dots\dots(2)$$

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where T = period in seconds

Knowing the deflected shape of the chimney due to inertia forces. Eq. [1] can be evaluated. For the shape function, any reasonable curve can be assumed. Even if the boundary conditions are not fully satisfied, the probability of getting a realistic value for the frequency is not unexpected. In this note two shape functions were investigated which are follows :

$$(1) \quad y = c x^2 \quad \dots\dots(3)$$

$$(2) \quad y = c [x^4 - 6 h^2 x^2] \quad \dots\dots(4)$$

where h = height of the chimney

c = a constant

x = height of any section from the base of the chimney.

It may be seen that the kinematic and most of the force boundary conditions are satisfied by these two polynomials. Therefore, it is anticipated that these polynomials will provide reasonable values for the frequency in accordance with Rayleigh's principle.

ANALYTICAL STUDY

Nineteen chimneys with height varying from 90 metres to 420 metres were selected. The properties of the chimneys are listed in Table 1. The period was computed using Eq. [3] and Eq [4] and the same is given in Table 2.

DISCUSSION OF ANALYSIS

It is seen that Eq [4] gives slightly higher values in the majority of the cases than Eq. [3]. Hence the solution given by Eq [4] may be considered to be closer to the true value. However, from Rayleigh's principle, it follows that the true period will be a little higher than the one given by Eq [4] or may be almost equal to the true value provided the curve resembles the exact dynamic deflection curve.

PROPOSED EQUATION FOR THE PERIOD

The Rayleigh method of computing the period is a tedious process. The sheer volume of computation involved in the procedure prohibits its use in the design office. Therefore, in the place of Rayleigh's rigorous solution, a simple equation is proposed as follows :

$$T = \frac{0.1813 H^2}{D \sqrt{E_0}} + \frac{H [HD]^{0.5}}{[47,000 - 15 H]} \dots\dots (5)$$

where

- T = period in seconds
- H = height of the chimney in metres
- D = outer base diameter in metres
- E_c = Young's modulus of concrete in MPa.

The prediction of Eq. [5] for the nineteen chimneys is shown in Table 2 along with the result of the Indian code equation. It is found that the IS 1893-1975 equation is somewhat involved and yields higher values consistently for all height of the chimneys.

SUMMARY

Fundamental period of a tapered chimney can be computed using the Rayleigh minimum principle. Because of lengthy computation which the principle entails, it is not suitable for ready use. Based on Rayleigh's solution, a simple equation is proposed for determining the period in a rapid manner. Prediction of the equation is in fair agreement with Rayleigh's principle in the majority of the cases. It is found that the equation is quite facile to work with.

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DEDICATION

This paper is dedicated to the patriot Mahakavi C. Subramania Bharathi whose works inspired the senior author to take up the task of this kind for the advancement of knowledge and progress of humanity.

REFERENCES

1. WIEGEL, R L.,
"Earthquake Engineering"
Prentice Hall, Inc., Inc., Englewood Cliffs, N.J 1970.
2. MARK FINTEL
"Hand book of Concrete Engineering"
Van Nostrand Reinhold Company, U S A 1974
3. Is 1893—1975 Indian Standard "Criteria for Earthquake Resistant Design of Structures"
New Delhi—1976.

Table 1 Properties of the Chimney

Sl. No.	Height in metres	Outer diameter at bottom in metres	Outer diameter at top in metres	Thickness at bottom in meters	Thickness at top in metres	Young's Modulus in MPa x 10 ⁴
1	91.34	8 13	3 02	0.61	0.15	2.434
2	104.71	10 06	5.03	0.46	0.23	2 399
3	107.44	8 36	5.23	0.61	0.15	2.405
4	127.10	11.45	6 45	0 46	0.15	2.599
5	137.16	12 19	5 41	0.61	0.18	2.144
6	162.76	10.68	5.69	0.91	0 23	2 420
7	180.00	15.00	6 00	0.40	0.22	2 720
8	200.00	16.66	7.00	0 45	0 25	2.720
9	220.00	18 00	7.50	0 60	0.27	2 720
10	240.00	20 00	8 00	0 55	0 30	2 720
11	260.00	21.50	9 00	0.60	0 35	2 720
12	280 00	23 00	10.50	0.65	0.40	2.720
13	300 00	25 00	11.00	0 70	0 45	2 720
14	320 00	27 00	12 00	0 80	0.50	2.720
15	340 00	30 00	15 00	0 90	0.60	2 720
16	360 00	34 00	18 00	1.00	0 75	2 720
17	380.00	36 00	20 00	1.10	0.90	2r720
18	400.00	38 00	21 50	1 25	1.00	2 720
19	420 00	40 00	23.00	1.40	1.10	2 720

Table 2 Prediction of Period in Sec. by Different Equations and Comparison

No.	Height of chimney in metres	Eq. [3]	Eq. [4]	Eq. [5]	IS 1893-1975 Equation
1	91.34	1.130	1.110	1.249	1.39
2	104.71	1.330	1.410	1.349	1.55
3	107.44	1.560	1.613	1.688	1.93
4	121.10	1.570	1.640	1.695	1.87
5	137.16	1.850	1.850	2.036	2.21
6	162.76	2.830	2.870	3.042	3.36
7	180.00	2.570	2.580	2.592	2.78
8	200.00	2.880	2.900	2.908	3.11
9	220.00	3.220	3.240	3.277	3.47
10	240.00	3.530	3.450	3.540	3.71
11	260.00	3.790	3.810	3.911	4.09
12	280.00	4.220	4.290	4.275	4.50
13	300.00	4.490	4.560	4.571	4.76
14	320.00	4.710	4.780	4.885	5.02
15	340.00	4.880	5.020	5.060	5.21
16	360.00	4.970	5.170	5.151	5.29
17	380.00	5.340	5.610	5.496	5.68
18	400.00	5.590	5.900	5.843	5.97
19	420.00	5.010	6.230	6.189	6.27